

# Convex Segmentochora

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preprint of August 2001, published in:  
Symmetry: Culture and Science, Vol. 11, Nos. 1-4, 139-181, 2000

## Abstract

Polytopes with all vertices both (A) on a (hyper-) sphere and (B) on a pair of parallel (hyper-) planes, and further (C) with all edges of equal length  $l$  will call segmentotopes. Moreover, in dimensions 2, 3 and 4 names like *segmentogon*, *segmentohedron*, and *segmentochoron* could be used. In this article the *convex* segmentotopes up to dimension 4 are listed.

## 1 Introduction

About 150 years of highdimensional research on polytopes have passed. The regular ones are well-known since those days: in 1852 L. Schlaefli completed his monograph on polyschemes. About 20 years after N. Johnson in 1966 had published the set of convex polyhedra with regular faces, Mrs. R. Blind had done the corresponding research in higher dimensions for polytopes with regular facets. The convex uniform ones of dimension 4 are readily listed on the webpages [http://member.aol.com/\\_ht\\_b/Polycell/uniform.html](http://member.aol.com/_ht_b/Polycell/uniform.html), and the complete list of all uniform ones of dimension 4 is still ongoing (J. Bowers and G. Olshevsky).

Sure, polychora, i.e. polytopes of dimension 4, are not so easy to visualize. This is especially due to the fact that for this attempt the 4th dimension has to be projected somehow into the span of the other 3 directions. One possibility, to do this, works rather well for figures with just one edge length. It shows the 4th dimension as a contraction. In this projection especially monostratic figures, i.e. figures with just one layer with respect to (at least) one direction, are easily illustrated by 2 concentric polyhedra, standing for the bottom and the top of the layer. The space inbetween will then be filled accordingly to the projection of the latter cells.

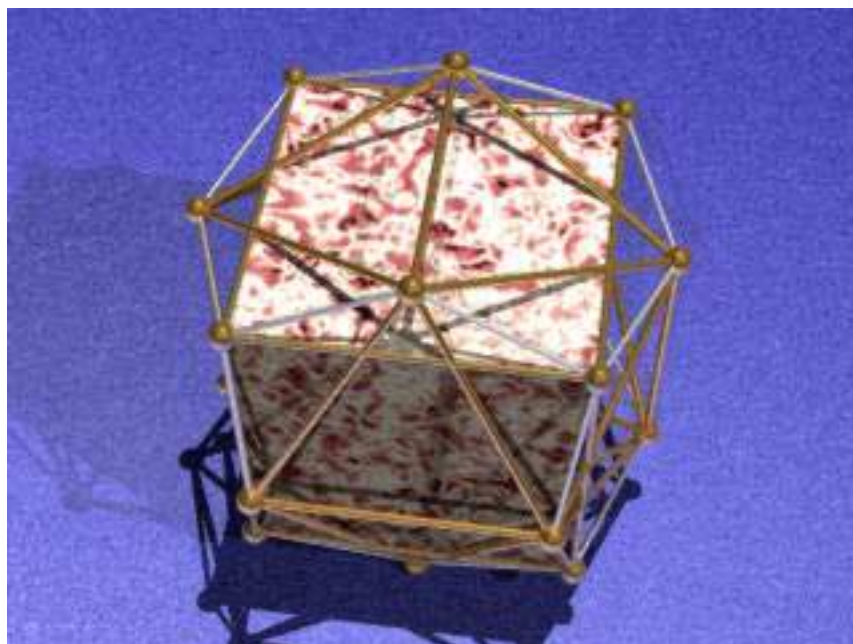
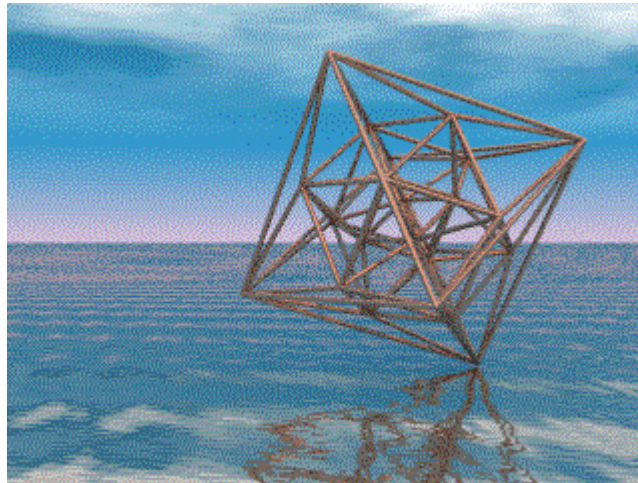


Figure 1: icosahedron atop cube

Figure 1 shows an example of such a projection of a segmentochoron. The 2 parallel polyhedra are a cube (solid) respectively an icosahedron (frame). Those edges of the latter which are parallel to the cube are joined to the faces of the cube by trigonal prisms. The vertices of the cube are joined to 8 of the icosahedral faces by tetrahedra. The remaining 12 icosahedral faces are joined to the still open squares of the trigonal prisms by square pyramids. Thus the cell count of that segmentochoron is: 8 tetrahedra + 12 square pyramids + 6 trigonal prisms + 1 cube + 1 icosahedron. In this Figure the arbitrary relative scaling was chosen such that the edges of cube and icosahedron do intersect in this projection. - Figure 1 was produced by Robert J. MacG. Dawson (robert.dawson@stmarys.ca).



**Figure 2: bistratic projection of an icositetrachoron:  
octahedron atop (pseudo) cuboctahedron atop octahedron**

Figure 2 was found on <http://www.math.tu-berlin.de/diskregeom/polymake/doc/polytope.gif>. It shows the skeleton of the regular icositetrachoron. In this bistratic projection it is visible as octahedron atop (pseudo) cuboctahedron atop octahedron. The equatorial cuboctahedron is marked as pseudo, as it is not a facet of the icositetrachoron; just as the equatorial square is not a face of the octahedron, which alike could be called 'point atop (pseudo) square atop point'. Nevertheless, both the inner and the outer half of the bistratic projected icositetrachoron are projections of valid segmentochora, which are monostratic. Then square faces of the cuboctahedron are joined to the vertices of the parallel octahedra by square pyramids (halves of octahedra) and the trigons of the cuboctahedron to the faces of the parallel octahedra by trigonal antiprisms (i.e. octahedra). Thus those segmentochora consist of 1+8 octahedra + 6 square pyramids + 1 cuboctahedron.

Polychora which are monostratic are the topic of this article. Especially we look at *convex segmentochora*. In general segmentotopes are defined to be polytopes (thereby following all implications thereof) and additionally have

- all vertices on a single hypersphere,
- all vertices on a (not necessarily unique) pair of parallel hyperplanes,
- all edges of unit length.

The first condition shows that the circumradius is well defined. Moreover, in union with condition 3 this implies that all faces have to be regular. Condition 2 implies that all edges, which don't lie completely within one of the hyperplanes, will join both, i.e. having one vertex each in either plane. Thence segmentotopes have to be monostratic. We restrict ourselves to *convex* segmentotopes, as their count grows rather fast with the dimension.

From this definition it follows that the top and bottom figures too are polytopes with all vertices on a single circumsphere. Thus, for convex segmentochora we have as possible top and bottom figures the following set:

Top or bottom figure	Circumradius
Point	0 (shear?)
Line	1/2 (shear?)
Trigon	$1/\sqrt{3} = 0.577350$ (shear?)
Square	$1/\sqrt{2} = 0.707107$ (shear?)
Pentagon	$\sqrt{(5+\sqrt{5})/10} = 0.850651$ (shear?)
Hexagon	1 (shear?)
Octagon	$\sqrt{(1+1/\sqrt{2})} = 1.306563$ (shear?)
Decagon	$(1+\sqrt{5})/2 = 1.618034$ (shear?)
N-gon: N>6,not 8,10	$1/(2*\sin(\pi/n))$ (shear?)
Tetrahedron	$\sqrt{3/8} = 0.612372$
Octahedron	$1/\sqrt{2} = 0.707107$
Cube	$\sqrt{3/4} = 0.866025$
Icosahedron	$\sqrt{(5+\sqrt{5})/8} = 0.951057$
Dodecahedron	$\sqrt{(9+3*\sqrt{5})/8} = 1.401259$
Cuboctahedron	1
Icosidodecahedron	$(1+\sqrt{5})/2 = 1.618034$
Truncated tetrahedron	$\sqrt{11/8} = 1.172604$
Truncated octahedron	$\sqrt{5/2} = 1.581139$
Truncated cube	$\sqrt{(7+4*\sqrt{2})/2} = 1.778824$
Truncated icosahedron	$\sqrt{(29+9*\sqrt{5})/8} = 2.478019$
Truncated dodecahedron	$\sqrt{(37+15*\sqrt{5})/8} = 2.969445$
Rhombicuboctahedron	$\sqrt{(5+\sqrt{8})/4} = 1.398966$
Rhombicosidodecahedron	$\sqrt{(\sqrt{5}+11/4)} = 2.232951$
Truncated cuboctahedron	$\sqrt{(13+6*\sqrt{2})/2} = 2.317611$
Truncated icosidodecahedron	$\sqrt{(31+12*\sqrt{5})/2} = 3.802394$
Snub cuboctahedron	$\sqrt{((1-\cos^2(x))/(3-4*\cos^2(x)))} = 1.343713$ [ $\cos(x) = (\text{cbrt}(1+\sqrt{11/27})+\text{cbrt}(1-\sqrt{11/27}))/\text{cbrt}(\sqrt{128}) = 0.842509$ ]
Snub icosidodecahedron	$\sqrt{((1-\cos^2(x))/(3-4*\cos^2(x)))} = 2.155837$ [ $\cos(x) = (\text{cbrt}(9+9*\sqrt{5})+\sqrt{102+162*\sqrt{5}})+\text{cbrt}(9+9*\sqrt{5}-\sqrt{102+162*\sqrt{5}}))/\text{cbrt}(288) = 0.857781$ ]
4-Pyramid (J1)	$1/\sqrt{2} = 0.707107$
5-Pyramid (J2)	$\sqrt{(5+\sqrt{5})/8} = 0.951057$
3-Cupola (J3)	1
4-Cupola (J4)	$\sqrt{(5+\sqrt{8})/4} = 1.398966$
5-Cupola (J5)	$\sqrt{(\sqrt{5}+11/4)} = 2.232951$
Rotunda (J6)	$(1+\sqrt{5})/2 = 1.618034$
Gyroelongated 5-pyramid (J11)	$\sqrt{(5+\sqrt{5})/8} = 0.951057$
Elongated 4-cupola (J19)	$\sqrt{(5+\sqrt{8})/4} = 1.398966$
Trigonal orthobicupola (J27)	1
Orthobiotunda (J34)	$(1+\sqrt{5})/2 = 1.618034$
Gyrated rhombicuboctahedron (J37)	$\sqrt{(5+\sqrt{8})/4} = 1.398966$
Metabidiminished icosahedron (J62)	$\sqrt{(5+\sqrt{5})/8} = 0.951057$
Tridiminished icosahedron (J63)	$\sqrt{(5+\sqrt{5})/8} = 0.951057$
Gyrated rhombicosidodecahedron (J72)	$\sqrt{(\sqrt{5}+11/4)} = 2.232951$
Parabigyrate rhombicosidodecahedron (J73)	$\sqrt{(\sqrt{5}+11/4)} = 2.232951$
Metabigyrate rhombicosidodecahedron (J74)	$\sqrt{(\sqrt{5}+11/4)} = 2.232951$
Trigyrate rhombicosidodecahedron (J75)	$\sqrt{(\sqrt{5}+11/4)} = 2.232951$
Diminished rhombicosidodecahedron (J76)	$\sqrt{(\sqrt{5}+11/4)} = 2.232951$
Diminished paragyrate rhombicosidodecahedron (J77)	$\sqrt{(\sqrt{5}+11/4)} = 2.232951$
Diminished metagyrate rhombicosidodecahedron (J78)	$\sqrt{(\sqrt{5}+11/4)} = 2.232951$
Diminished bigyrate rhombicosidodecahedron (J79)	$\sqrt{(\sqrt{5}+11/4)} = 2.232951$
Parabidiminished rhombicosidodecahedron (J80)	$\sqrt{(\sqrt{5}+11/4)} = 2.232951$
Metabidiminished rhombicosidodecahedron (J81)	$\sqrt{(\sqrt{5}+11/4)} = 2.232951$
Metabidiminished gyrated rhombicosidodecahedron (J82)	$\sqrt{(\sqrt{5}+11/4)} = 2.232951$
Tridiminished rhombicosidodecahedron (J83)	$\sqrt{(\sqrt{5}+11/4)} = 2.232951$
3-Prism	$\sqrt{7/12} = 0.763763$
5-Prism	$\sqrt{(15+2*\sqrt{5})/20} = 0.986715$
6-Prism	$\sqrt{5}/2 = 1.118034$

Top or bottom figure	Circumradius
8-Prism	$\sqrt{(5+\sqrt{8})/4} = 1.398966$
10-Prism	$\sqrt{(7+2*\sqrt{5})/4} = 1.693527$
N-Prism: N>6, not 8, 10	$\sqrt{(1+\csc^2(\pi/n))/2}$
4-Antiprism	$\sqrt{(4+\sqrt{2})/8} = 0.822664$
5-Antiprism	$\sqrt{(5+\sqrt{5})/8} = 0.951057$
6-Antiprism	$\sqrt{(3+\sqrt{3})/4} = 1.087664$
8-Antiprism	$\sqrt{((3-\sqrt{2+\sqrt{2}})/(8-4*\sqrt{2+\sqrt{2}}))} = 1.375549$
10-Antiprism	$\sqrt{((3*\sqrt{2}-\sqrt{5+\sqrt{5}})/(8*\sqrt{2}-4*\sqrt{5+\sqrt{5}}))} = 1.674505$
N-Antiprism: N>6, not 8, 10	$\sqrt{((3-2*\cos(\pi/n))/(8-8*\cos(\pi/n)))}$

Table 1: list of possible top and bottom facets and their circumradii

Further it follows from the definition that the laterals have to be segmentotopes in turn. So, in order to give a list of all segmentochora one has to look first at the possibilities for segmentogons and segmentohedra. In the convex cases we have (arrow means ‘atop’):

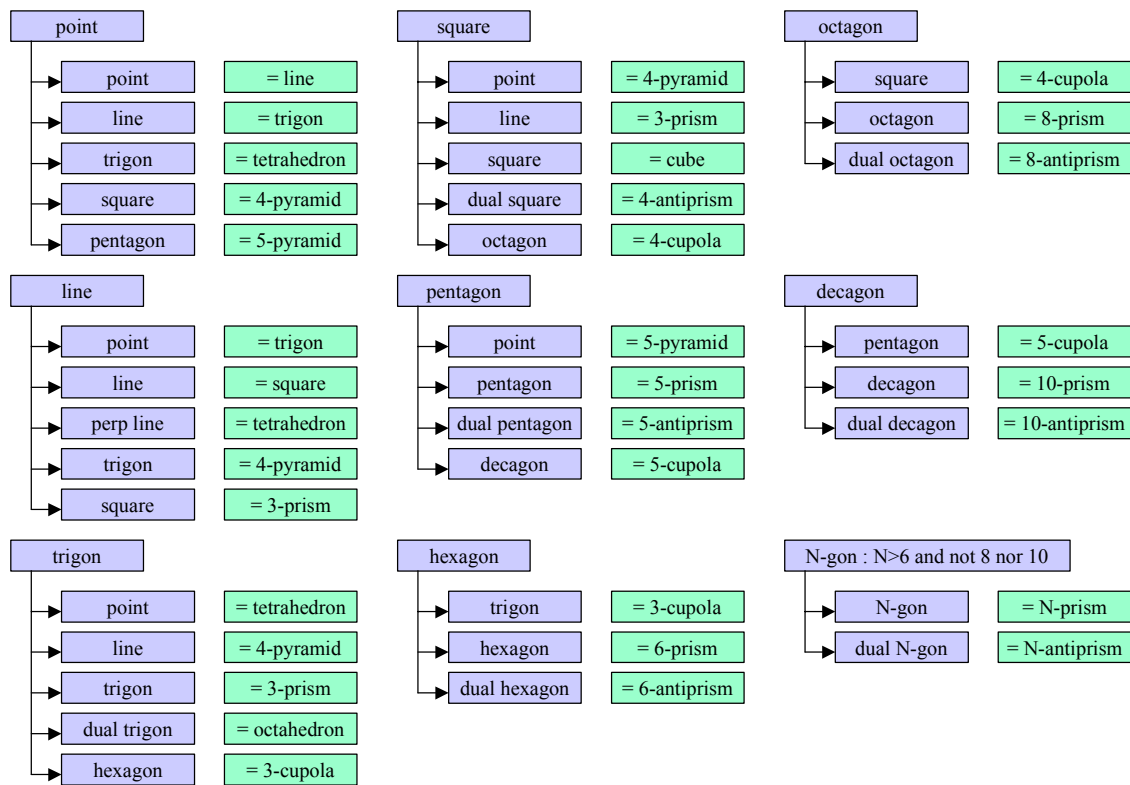


Figure 3: low-dimensional convex segmentotopes: x atop y

Maybe some first intuitive examples are in place. The first set of segmentochora clearly is that of 4D prisms. Take any polyhedron from Table 1, erect on its faces ordinary 3D prisms, bend it into the fourth dimension such that the lateral squares will meet, and close that figure with a second copy of the starting polyhedron: "x || x".

A second set of likewise trivial segmentochora is that of 4D pyramids. Take any polyhedron from Table 1 which has a circumradius < 1, put an additional vertex along the fourth

dimension atop it such that all polyhedral vertices are one unit apart: "point || x". The lateral facets are 3D pyramids on top of the faces of the bottom polyhedron x.

More interesting segmentochora are constructable from the pyramidal subgroups of symmetry groups  $[[n,m,2]]$ . For convexity take  $(n,m) = (3,3), (3,4)$  or  $(3,5)$  (but table 2 applies to  $(5/2,3)$  and  $(5/2,5)$  too). Take 2 convex uniform polyhedra of some group  $[[n,m]]$ , place them symmetrically atop another, and, if their circumradii do not differ too much, the result will be a valid segmentochoron again. - The margin of Table 2 gives the top and bottom polyhedra of the segmentochora in truncation-notation of Coxeter-Schlaefli symbols (numbers behind the 't' are positions of ringed knots in the Coxeter-Dynkin diagram). The body lists the additional, i.e. lateral facets.

	$t0\{n,m\}$	$t1\{n,m\}$	$t2\{n,m\}$	$t01\{n,m\}$	$t02\{n,m\}$	$t12\{n,m\}$	$t012\{n,m\}$
$t0\{n,m\}$	n-p	n-ap, m-pyr	n-pyr, tet, m-pyr	n-cup, m-pyr	n-p, 3p, m-pyr	n-ap, tet, 2m-pyr	n-cup, 3p, 2m-pyr
$t1\{n,m\}$		n-p, m-p	n-pyr, m-ap	n-cup, m-p	n-ap, 4pyr, m-ap	n-p, m-cup	n-cup, 4pyr, m-cup
$t2\{n,m\}$			m-p	2n-pyr, tet, m-ap	n-pyr, 3p, m-p	n-pyr, m-cup	2n-pyr, 3p, m-cup
$t01\{n,m\}$				2n-p, m-p	n-cup, 3p, m-ap	n-cup, tet, m-cup	2n-p, 3p, m-cup
$t02\{n,m\}$					n-p, cube, m-p	n-ap, 3p, m-cup	n-cup, cube, m-cup
$t12\{n,m\}$						n-p, 2m-p	n-cup, 3p, 2m-p
$t012\{n,m\}$							2n-p, cube, 2m-p

Table 2: lateral facets of segmentochora with axial symmetry from  $[[n,m]]$

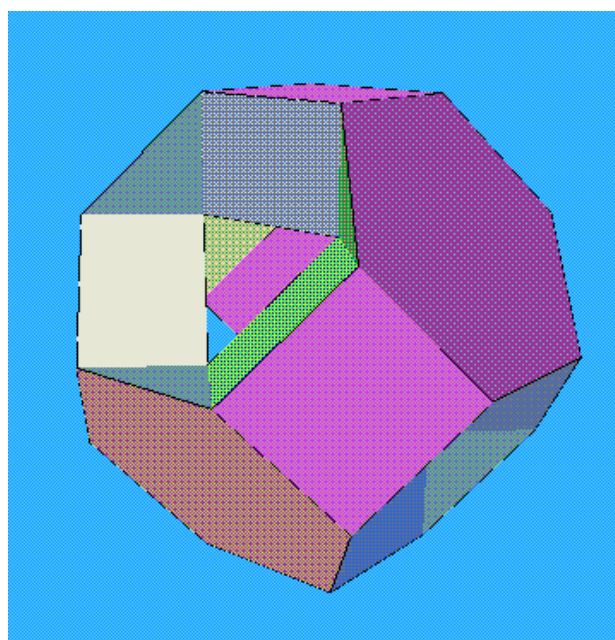


Figure 4: cuboctahedron atop truncated octahedron

Figure 4 shows an projection of the cuboctahedron atop truncated octahedron, which is an example of Table 2 (it visualizes both  $t1\{3,4\} \parallel t01\{3,4\}$  and  $t02\{3,3\} \parallel t012\{3,3\}$ ). It was produced from data of Alex Doskey, at LSUHSC of the Louisiana State University. Therin half of the triangular cupolae are removed together with the inner cuboctahedron, in order to get the inner structure visible.

Even more generall one will have to take any 2 figures from Table 1 in any possible relative orientation and has to decide whether there would be a convex segmentochoron lying in between, i.e. whether vertices could be joined by unit edges in such a way, that the lateral facets would be from the list of Figure 3 only. This task for the 4 dimensional set has be done by the author manually within the span of summer 2000 to summer 2001. Although he has no firm proof, the author supposes the list to be complete:

The circumradius (R) of a segmentotope is readily accessible from the circumradii of its top and bottom facets ( $r1, r2$ ), the height (H) between them and (if those facets would be lower dimensional) the shear ( $S1, S2$ ) of their centers parallel to those hyperplanes (taken perpendicular to one another), see Figure 5. It is given by  $4 \cdot R^2 \cdot H^2 = ((r2^2 + S2^2) - (r1^2 + S1^2))^2 + 2 \cdot ((r1^2 + S1^2) + (r2^2 + S2^2)) \cdot H^2 + H^4$ . (The easiest example for a non-vanishing shear is the square pyramid, looked at as a trigonal wedge, i.e. a line atop a trigon: the center of the line is not directly above the center of the trigon.) Clearly, due to the existance of the circum-hypersphere, non-vanishing shears are possible only for subdimensional top or bottom facets.

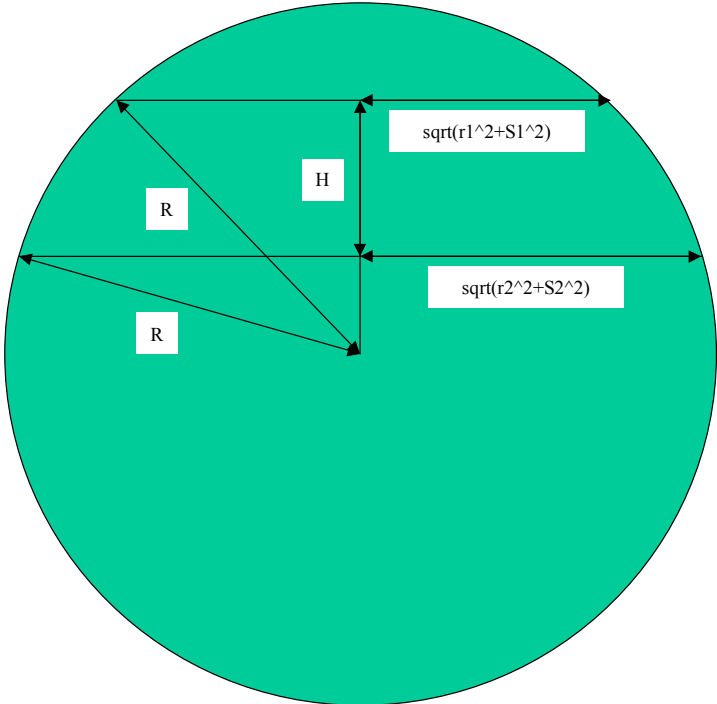


Figure 5: Getting the circumradius R as function of H, r1, r2, S1, S2

For the extrapolation of names of polyhedra to names of polychora some remarks are usefull. The name "**antiprism**" will be used in analogy to the 3D case whenever top and bottom facet

are vice versas duals. Thence only for selfdual top facets (and therefore bottom facets as well) those 2 facets are congruent, as they are for 3D antiprisms. Names will be given like this:  $\langle \text{top-facet} \rangle$ -*antiprism* or equivalently  $\langle \text{bottom-facet} \rangle$ -*antiprism*. Those are the cases  $t0\{n,m\} \parallel t2\{n,m\}$  from Table 2.

The observation, that for 3D cupolas the bottom face is up to scaling the kernel of intersection of a dual pair of the top face, leads to a first extension of this name to polychora: If the top facet is a regular polyhedron ( $t0\{n,m\}$ ), the bottom facet ought to be the corresponding quasiregular one (i.e. the rectified polyhedron,  $t1\{n,m\}$ ). Note that the corresponding faces  $\{n\}$  of those are relatively rotated, they have to be joined by antiprisms, which in turn generalize the squares of 3D cupolas. Here the name "**cupola**" will further be used within 4D for all those segmentotopes, where the lateral facets are pyramids and antiprisms only. Names will be all like this:  $\langle \text{top-facet} \rangle$ -*cupola*.

Note that there could be a possible other extrapolation of cupolae as well by generalizing the lateral squares to prisms. This would imply for regular top facets ( $t0\{n,m\}$ ) the bottom facets to be the corresponding rhombi forms (i.e. the runcinated polyhedron,  $t02\{n,m\}$ ). The lateral elements would then be pyramids, prisms, and trigonal prisms (seen as digonal cupolas). – But such a definition does not even apply to all possible regular top facets, for vertices of the icosahedron cannot be joined to those of the rhombicosidodecahedron using only edges of unit length, even by bending into 4D. But this does extrapolate cupolae from being monostratic cups of uniform polyhedra to monostratic cups of uniform polychora (top facets  $t0\{n,m\}$  imply the sectioned polychoron to be  $t03\{3,m,n\}$ .) Using this last observation, in here those few segmentochora are called  $\langle \text{polychoron} \rangle$ -*s*  $\langle \text{top-facet} \rangle$ -*cup*.

Names like "**pyramids**" and "**prisms**" extend unambiguously to higher dimensions, meaning polytopes which are 'point atop facet' resp. 'facet atop (the same ungyrated) facet'. Names will be built like this:  $\langle \text{bottom-facet} \rangle$ -*pyramid* resp.  $\langle \text{top-facet} \rangle$ -*prism*. – Note that only those pyramids are selfdual, where their bottom facet is selfdual in turn.

Finally "**wedges**" are defined as those segmentotopes where the top facet is subdimensional and is moreover a facet of the bottom facet. As in 3D the facets of faces are edges only, it is enough to mention the bottom face (a square-wedge is a trigonal prisms, standing on its square; a trigon-wedge is a square-pyramid, standing on its trigon). In 4D the bottom polyhedron might have different faces, thence the names are set up like  $[\langle \text{top facet} \rangle$ -*al*]  $\langle \text{bottom-facet} \rangle$ -*al wedge*. (Remind that the top facet is subdimensional.)

After these conventions the rest of this article is devoted to the explicit list of convex segmentotopes up to dimension 4. The headers are given each in the form "x || y" which is to be read as " $\langle \text{top facet} \rangle$  atop  $\langle \text{bottom facet} \rangle$ ". Within each symbol, x and y are in the body of this article chosen to be of ascending dimension, and, if of equal dimension, to be of ascending (facetal) circumradius. The whole list is sorted by ascending (full dimensional) circumradius, i.e polychoral curvature. Within the realm of equal circumradii they are sorted by descending height, and, if equal, the degree of gyration and diminuation is chosen to be ascending. Different views of the same segmentochoron are grouped together, sorted by descending height, and if necessary thereafter by ascending circumradii of the top facets.

Note that we distinguish for polygons, prisms and antiprisms the cases  $N = (2,)$  3, 4, 5, 6, 8, and 10 from the others. This was done because of the possible interference with the other polyhedra of Table 1, respectively its impossibility.

## 2 Dimensional

### 2.1 point || line

*height:*  $\sqrt{3/4} = 0.866025$   
*circumradius:*  $\sqrt{1/3} = 0.577350$   
*other names:* regular trigon  
*comments:* selfdual, regular

### 2.2 line || line

*height:* 1  
*circumradius:*  $\sqrt{1/2} = 0.707107$   
*other names:* square  
*comments:* selfdual, regular

## 3 Dimensional

### 3.1 point || trigon

*height:*  $\sqrt{2/3} = 0.816497$   
*shear:* 0

#### 3.1.1 line || perpendicular line

*height:*  $\sqrt{1/2} = 0.707107$   
*shear (top):* 0  
*shear (bottom):* 0  
*circumradius:*  $\sqrt{3/8} = 0.612372$   
*other names:* tetrahedron, trigonal pyramid, digonal antiprism  
*comments:* selfdual, regular  
*faces:* 4 trigons

### 3.2 trigon || dual trigon

*height:*  $\sqrt{2/3} = 0.816497$   
*circumradius:*  $\sqrt{1/2} = 0.707107$   
*other names:* octahedron, trigonal antiprism  
*comments:* regular  
*faces:* 8 trigons

### 3.3 line || trigon

*height:*  $\sqrt{2/3} = 0.816497$   
*shear:*  $1/\sqrt{12} = 0.288675$

#### 3.3.1 point || square

*height:*  $\sqrt{1/2} = 0.707107$   
*shear:* 0  
*circumradius:*  $1/\sqrt{2} = 0.707107$   
*other names:* square pyramid, J1, half of octahedron, trigonal wedge  
*comments:* selfdual  
*faces:* 4 trigons + 1 square

### 3.4 trigon || trigon

*height:* 1

### 3.4.1 line || square

*height:*  $\sqrt{3/4} = 0.866025$   
*shear:* 0  
*circumradius:*  $\sqrt{7/12} = 0.763763$   
*other names:* trigonal prism, digonal cupola, tetragonal wedge  
*comments:* uniform  
*faces:* 2 trigons + 3 squares

### 3.5 square || dual square

*height:*  $1/\sqrt{\sqrt{2}} = 0.840896$   
*circumradius:*  $\sqrt{(4+\sqrt{2})/8} = 0.822664$   
*other names:* square antiprism  
*comments:* uniform  
*faces:* 8 trigons + 2 squares

### 3.6 square || square

*height:* 1  
*circumradius:*  $\sqrt{3/4} = 0.866025$   
*other names:* cube, hexahedron, square prism  
*comments:* regular  
*faces:* 6 squares

### 3.7 pentagon || dual pentagon

*height:*  $\sqrt{(5+\sqrt{5})/10} = 0.850651$   
*circumradius:*  $\sqrt{(5+\sqrt{5})/8} = 0.951057$   
*other names:* pentagonal antiprism, parabidiminished icosahedron  
*comments:* uniform  
*faces:* 10 trigons + 2 pentagons

### 3.8 point || pentagon

*height:*  $\sqrt{(5-\sqrt{5})/10} = 0.525731$   
*shear:* 0  
*circumradius:*  $\sqrt{(5+\sqrt{5})/8} = 0.951057$   
*other names:* pentagonal pyramid, J2  
*comments:* selfdual, kind of diminished icosahedron  
*faces:* 5 trigons + 1 pentagon

### 3.9 pentagon || pentagon

*height:* 1  
*circumradius:*  $\sqrt{(15+2*\sqrt{5})/20} = 0.9867151$   
*other names:* pentagonal prism  
*comments:* uniform  
*faces:* 5 squares + 2 pentagons

### 3.10 trigon || hexagon

*height:*  $\sqrt{2/3} = 0.816497$   
*circumradius:* 1  
*other names:* trigonal cupola, J3, half of cuboctahedron  
*faces:* 1+3 trigons + 3 squares + 1 hexagon



### 3.11 hexagon || dual hexagon

*height:*  $\sqrt{\sqrt{3}-1} = 0.855600$   
*circumradius:*  $\sqrt{(3+\sqrt{3})/4} = 1.087664$   
*other names:* hexagonal antiprism  
*comments:* uniform  
*faces:* 12 trigons + 2 hexagons

### 3.12 hexagon || hexagon

*height:* 1  
*circumradius:*  $\sqrt{5}/2 = 1.118034$   
*other names:* hexagonal prism  
*comments:* uniform  
*faces:* 6 squares + 2 hexagons

### 3.13 octagon || dual octagon

*height:*  $\sqrt{(1+\sqrt{2+\sqrt{2}})/(2+\sqrt{2+\sqrt{2}}))} = 0.860296$   
*circumradius:*  $\sqrt{(3-\sqrt{2+\sqrt{2}})/(8-4*\sqrt{2+\sqrt{2}}))} = 1.375549$   
*other names:* octagonal antiprism  
*comments:* uniform  
*faces:* 16 trigons + 2 octagons

### 3.14 octagon || octagon

*height:* 1  
*circumradius:*  $\sqrt{(5+2*\sqrt{2})/4} = 1.398966$   
*other names:* octagonal prism, bidiminshed rhombicuboctahedron  
*comments:* uniform  
*faces:* 8 squares + 2 octagons

### 3.15 square || octagon

*height:*  $\sqrt{1/2} = 0.707107$   
*circumradius:*  $\sqrt{(5+2*\sqrt{2})/4} = 1.398966$   
*other names:* tetragonal cupola, J4  
*comments:* kind of diminished rhombicuboctahedron  
*faces:* 4 trigons + 1+4 squares + 1 octagon

### 3.16 decagon || dual decagon

*height:*  $\sqrt{(1+\sqrt{(5+\sqrt{5})/2})/(2+\sqrt{(5+\sqrt{5})/2}))} = 0.862397$   
*circumradius:*  $\sqrt{(3-\sqrt{(5+\sqrt{5})/2})/(8-4*\sqrt{(5+\sqrt{5})/2}))} = 1.674505$   
*other names:* decagonal antiprism  
*comments:* uniform  
*faces:* 20 trigons + 2 decagons

### 3.17 decagon || decagon

*height:* 1  
*circumradius:*  $\sqrt{(7+2*\sqrt{5})/4} = 1.693527$   
*other names:* decagonal prism  
*comments:* uniform  
*faces:* 10 squares + 2 decagons

### 3.18 pentagon || decagon

*height:*  $\sqrt{(5-\sqrt{5})/10} = 0.525731$   
*circumradius:*  $\sqrt{(\sqrt{5}+1)/4} = 2.232951$   
*other names:* pentagonal cupola, J5  
*comments:* kind of diminished rhombicosidodecahedron  
*faces:* 5 trigons + 5 squares + 1 pentagon + 1 decagon

### 3.19 n-gon || dual n-gon ( $n \neq 2, 3, 4, 5, 6, 8, 10$ )

*height:*  $\sqrt{(1+2*\cos(\pi/n))/(2+2*\cos(\pi/n))}$   
*circumradius:*  $\sqrt{(3-2*\cos(\pi/n))/(8-8*\cos(\pi/n))}$   
*other names:* n-gonal antiprism  
*comments:* uniform  
*faces:*  $2*n$  trigons + 2 n-gons

### 3.20 n-gon || n-gon ( $n \neq 3, 4, 5, 6, 8, 10$ )

*height:* 1  
*circumradius:*  $\sqrt{(1+\csc^2(\pi/n))/2}$   
*other names:* general n-gonal prism  
*comments:* uniform  
*faces:*  $n$  squares + 2 n-gons

## 4 Dimensional

### 4.1 point || tetrahedron

*height:*  $\sqrt{5/8} = 0.790569$   
*shear:* 0

#### 4.1.1 line || perpendicular trigon

*height:*  $\sqrt{5/12} = 0.645497$   
*shear (top):* 0  
*shear (bottom):* 0  
*circumradius:*  $\sqrt{2/5} = 0.632456$   
*other names:* pentachoron  
*comments:* regular, selfdual  
*cells:* 5 tetrahedra

### 4.2 tetrahedron || dual tetrahedron

*height:*  $1/\sqrt{2} = 0.707107$   
*circumradius:*  $1/\sqrt{2} = 0.707107$   
*other names:* hexadecachoron, tetrahedral antiprism  
*comments:* regular  
*cells:* 16 tetrahedra

### 4.3 point || octahedron

*height:*  $1/\sqrt{2} = 0.707107$   
*shear:* 0

#### 4.3.1 trigon || gyrated tetrahedron

*height:*  $1/\sqrt{2} = 0.707107$   
*shear:*  $1/\sqrt{24} = 0.204124$

*circumradius:*  $1/\sqrt{2} = 0.707107$   
*other names:* octahedral pyramid, half of hexadecachoron  
*comments:* homohedral  
*cells:* 8 tetrahedra + 1 octahedron

#### 4.4 point || square pyramid

*height:*  $1/\sqrt{2} = 0.707107$   
*shear:* 0

##### 4.4.1 line || tetrahedron

*height:*  $1/\sqrt{2} = 0.707107$   
*shear:*  $1/\sqrt{8} = 0.353553$

##### 4.4.2 trigon || inclined trigon

*height:*  $1/\sqrt{2} = 0.707107$   
*shear (top):*  $1/\sqrt{24} = 0.204124$   
*shear (bottom):*  $1/\sqrt{24} = 0.204124$

##### 4.4.3 line || perpendicular square

*height:* 1/2  
*shear (top):* 0  
*shear (bottom):* 0  
*circumradius:*  $1/\sqrt{2} = 0.707107$   
*other names:* square-pyramidal pyramid, quarter of hexadecachoron  
*comments:* selfdual  
*cells:* 4 tetrahedra + 2 square pyramids

#### 4.5 tetrahedron || octahedron

*height:*  $\sqrt{5/8} = 0.790569$   
*circumradius:*  $\sqrt{3/5} = 0.774597$   
*other names:* rectified pentachoron, tetrahedral cupola  
*comments:* uniform, homohedral  
*cells:* 5 tetrahedra + 5 octahedra

#### 4.6 tetrahedron || square pyramid

*height:*  $\sqrt{5/8} = 0.790569$

##### 4.6.1 trigon || octahedron

*height:*  $\sqrt{5/8} = 0.790569$   
*shear:*  $1/\sqrt{24} = 0.204124$

##### 4.6.2 trigon || gyrated trigonal prism

*height:*  $\sqrt{5/12} = 0.645497$   
*shear:* 0  
*circumradius:*  $\sqrt{3/5} = 0.774597$   
*other names:* octahedral wedge  
*comments:* kind of diminished rectified-pentachoron (trigon as "tetrahedron - tetrahedron" and octahedron as "octahedron - trigon")  
*cells:* 3 tetrahedra + 2 octahedra + 3 square pyramids + 1 trigonal prism

#### 4.7 line || square pyramid

*height:*  $\sqrt{5/8} = 0.790569$   
*shear:*  $1/\sqrt{8} = 0.353553$

##### 4.7.1 trigon || tetrahedron

*height:*  $\sqrt{5/8} = 0.790569$   
*shear:*  $1/\sqrt{6} = 0.408248$

##### 4.7.2 point || trigonal prism

*height:*  $\sqrt{5/12} = 0.645497$   
*shear:* 0

##### 4.7.3 trigon || orthogonal square (2 square-edges parallel to 1 trigon-edge)

*height:*  $\sqrt{5/12} = 0.645497$   
*shear (top):* 0  
*shear (bottom):*  $1/\sqrt{12} = 0.288675$   
*circumradius:*  $\sqrt{3/5} = 0.774597$   
*other names:* trigonal-prismatic pyramid, tetrahedral wedge  
*comments:* kind of diminished rectified-pentachoron (tetrahedron as "tetrahedron - trigon" and trigon as „octahedron - octahedron“)  
*cells:* 2 tetrahedra + 3 square pyramids + 1 trigonal prism

#### 4.8 trigon || square pyramid

*height:*  $\sqrt{5/8} = 0.790569$   
*shear:*  $1/\sqrt{24} = 0.204124$

##### 4.8.1 square || tetrahedron

*height:*  $\sqrt{5/8} = 0.790569$   
*shear:* 0

##### 4.8.2 line || orthogonal trigonal prism

*height:*  $\sqrt{5/12} = 0.645497$   
*shear:*  $1/\sqrt{12} = 0.288675$   
*circumradius:*  $\sqrt{3/5} = 0.774597$   
*other names:* trigonal square-pyramidal wedge  
*comments:* kind of bidiminished rectified-pentachoron (tetrahedron as "tetrahedron - 2 edges" and square as "octahedron - 2 square pyramids")  
*cells:* 1 tetrahedron + 4 square pyramids + 2 trigonal prisms

#### 4.9 tetrahedron || tetrahedron

*height:* 1

##### 4.9.1 line || parallel trigonal prism

*height:*  $\sqrt{2/3} = 0.816497$   
*shear:* 0

##### 4.9.2 square || orthogonal square

*height:*  $1/\sqrt{2} = 0.707107$   
*shear (top):* 0

*shear (bottom):* 0  
*circumradius:*  $\sqrt{5/8} = 0.790569$   
*other names:* tetrahedral prism  
*comments:* uniform  
*cells:* 2 tetrahedra + 4 trigonal prisms

#### 4.10 trigon || trigonal prism

*height:*  $\sqrt{3/4} = 0.866025$   
*shear:* 0  
*circumradius:*  $\sqrt{2/3} = 0.816497$   
*other names:* trigon-trigon-diprism, direct sum of 2 trigons, trigonal trigonal-prismatic wedge  
*comments:* uniform, isochoric  
*cells:* 6 trigonal prisms

#### 4.11 octahedron || octahedron

*height:* 1

##### 4.11.1 trigonal prism || gyrated trigonal prism

*height:*  $\sqrt{2/3} = 0.816497$   
*circumradius:*  $\sqrt{3/4} = 0.866025$   
*other names:* octahedral prism  
*comments:* uniform  
*cells:* 2 octahedra + 8 trigonal prisms

#### 4.12 square pyramid || square pyramid

*height:* 1

##### 4.12.1 square || trigonal prism

*height:*  $\sqrt{2/3} = 0.816497$   
*shear:*  $1/\sqrt{12} = 0.288675$

##### 4.12.2 line || cube

*height:*  $\sqrt{1/2} = 0.707107$   
*shear:* 0  
*circumradius:*  $\sqrt{3/4} = 0.866025$   
*other names:* square-pyramidal prism, square trigonal-prismatic wedge  
*comments:* diminished octahedral-prism (twice square pyramid as "octahedron - square pyramid")  
*cells:* 2 square pyramids + 4 trigonal prisms + 1 cube

#### 4.13 trigonal prism || reflected orthogonal trigonal prism

*height:*  $\sqrt{2/3} = 0.816497$   
*circumradius:*  $\sqrt{3/4} = 0.866025$   
*other names:* -  
*comments:* kind of gyrated octahedral-prism (as 2 square-pyramidal prisms (see 4.12) gyro-joined at a cube)  
*cells:* 4 square pyramids + 4+4 trigonal prisms

#### 4.14 square || square antiprism

*height:*  $\sqrt{4-\sqrt{2}}/2 = 0.804019$   
*shear:*  $(\sqrt{2}-1)/\sqrt{\sqrt{32}} = 0.174155$

##### 4.14.1 square || gyrated cube

*height:*  $\sqrt{\sqrt{8}-1}/2 = 0.676097$   
*shear:* 0  
*circumradius:*  $\sqrt{(4+\sqrt{2})/7} = 0.879465$   
*other names:* square square-antiprismatic wedge  
*comments:* kind of bidiminished cubic-antiprism (square as "octahedron - 2 square pyramids" and cube as "cube - 2 squares")  
*cells:* 4 tetrahedra + 4 square pyramids + 2 square antiprisms + 1 cube

#### 4.15 octahedron || cube

*height:*  $\sqrt{\sqrt{8}-1}/2 = 0.676097$   
*circumradius:*  $\sqrt{(4+\sqrt{2})/7} = 0.879465$   
*other names:* octahedral antiprism, cubic antiprism  
*cells:* 8+12 tetrahedra + 1 octahedron + 6 square pyramids + 1 cube

#### 4.16 square pyramid || gyrated cube

*height:*  $\sqrt{\sqrt{8}-1}/2 = 0.676097$   
*circumradius:*  $\sqrt{(4+\sqrt{2})/7} = 0.879465$   
*other names:* -  
*comments:* kind of diminished cubic-antiprism (square pyramid as "octahedron - square pyramid" and cube as "cube - square")  
*cells:* 4+4+4 tetrahedra + 1+1+4 square pyramids + 1 square antiprism + 1 cube

#### 4.17 square || gyrated square pyramid

*height:*  $\sqrt{\sqrt{8}-1}/2 = 0.676097$   
*shear:* 1/2

##### 4.17.1 point || square antiprism

*height:*  $\sqrt{(4-\sqrt{2})/8} = 0.568527$   
*shear:* 0  
*circumradius:*  $\sqrt{(4+\sqrt{2})/7} = 0.879465$   
*other names:* square-antiprismatic pyramid  
*comments:* kind of diminished cubic-antiprism (square as "cube - cube" and square pyramid as "octahedron - square pyramid")  
*cells:* 8 tetrahedra + 2 square pyramids + 1 square antiprism

#### 4.18 trigonal prism || trigonal prism

*height:* 1

##### 4.18.1 square || cube

*height:*  $\sqrt{3/4} = 0.866025$   
*shear:* 0

*circumradius:*  $\sqrt{5/6} = 0.912871$   
*other names:* trigon-square-diprism, direct sum of trigon and square, trigonal-prismatic prism, cubic wedge  
*comments:* uniform  
*cells:* 4 trigonal prisms + 3 cubes

#### 4.19 square antiprism || square antiprism

*height:* 1

##### 4.19.1 cube || gyrated cube

*height:*  $\sqrt{\sqrt{1/2}} = 0.840896$   
*circumradius:*  $\sqrt{(6+\sqrt{2})/8} = 0.962692$   
*other names:* square-antiprismatic prism  
*comments:* uniform  
*cells:* 8 trigonal prisms + 2 square antiprisms + 2 cubes

#### 4.20 cube || cube

*height:* 1  
*circumradius:* 1  
*other names:* tesseract, hypercube, octachoron, square-square-diprism, cubic prism  
*comments:* regular  
*cells:* 8 cubes

#### 4.21 cube || icosahedron

*height:*  $(1+\sqrt{5})/4 = 0.809017$   
*circumradius:* 1  
*other names:* -  
*cells:* 8 tetrahedra + 12 square pyramids + 6 trigonal prisms + 1 cube + 1 icosahedron

#### 4.22 pentagon || pentagonal antiprism

*height:*  $(1+\sqrt{5})/4 = 0.809017$   
*shear:*  $\sqrt{(5-2*\sqrt{5})/20} = 0.162460$

##### 4.22.1 pentagon || gyrated pentagonal prism

*height:*  $\sqrt{(5+2*\sqrt{5})/20} = 0.688191$   
*shear:* 0  
*circumradius:* 1  
*other names:* pentagonal pentagonal-antiprismatic wedge  
*cells:* 5 tetrahedra + 5 square pyramids + 2 pentagonal antiprisms + 1 pentagonal prism

#### 4.23 tetrahedron || cuboctahedron

*height:*  $\sqrt{5/8} = 0.790569$   
*circumradius:* 1  
*other names:* half of runcinated pentachoron, half of small prismaticodecachoron  
*cells:* 1+4 tetrahedra + 4+6 trigonal prisms + 1 cuboctahedron

#### 4.24 tetrahedron || trigonal cupola

*height:*  $\sqrt{5/8} = 0.790569$   
*circumradius:* 1  
*comments:* kind of diminished half-of-runcinated-pentachoron (tetrahedron as "tetrahedron - trigon" and trigonal cupola as "cuboctahedron - trigonal cupola")  
*cells:* 2 tetrahedra + 6 trigonal prisms + 2 trigonal cupolae

#### 4.25 trigon || trigonal cupola

*height:*  $\sqrt{5/8} = 0.790569$   
*shear:*  $1/\sqrt{24} = 0.204124$

##### 4.25.1 hexagon || trigonal prism

*height:*  $\sqrt{5/12} = 0.645497$   
*shear:* 0  
*circumradius:* 1  
*other names:* trigonal trigonal-cupolaic wedge  
*comments:* kind of diminished half-of-runcinated-pentachoron (trigon as "tetrahedron - tetrahedron" and trigonal cupola as "cuboctahedron - trigonal cupola")  
*cells:* 3 tetrahedra + 1+3 trigonal prism + 2 trigonal cupolae

#### 4.26 square || square pyramid

*height:*  $1/\sqrt{2} = 0.707107$   
*shear:*  $1/\sqrt{2} = 0.707107$

##### 4.26.1 point || cube

*height:* 1/2  
*shear:* 0  
*circumradius:* 1  
*other names:* cubic pyramid, tetragonal square-pyramidal wedge  
*comments:* kind of diminished octahedral-cupola (square as "cuboctahedron - cuboctahedron" and square pyramid as "octahedron - square pyramid")  
*cells:* 6 square pyramids + 1 cube

#### 4.27 trigon || gyrated trigonal cupola

*height:*  $1/\sqrt{2} = 0.707107$   
*shear:*  $1/\sqrt{6} = 0.408248$

##### 4.27.1 hexagon || octahedron

*height:*  $1/\sqrt{2} = 0.707107$   
*shear:* 0  
*circumradius:* 1  
*other names:* -  
*comments:* kind of (bi-)diminished octahedral-cupola (trigon as "octahedron - octahedron" and trigonal cupola as "cuboctahedron - trigonal cupola" -resp.- octahedron as "octahedron - 2 trigons" and hexagon as "cuboctahedron - 2 trigonal cupolae")

*cells:* 1 octahedron + 6 square pyramids  
+ 2 trigonal cupolae

#### **4.28 square || cuboctahedron**

*height:*  $1/\sqrt{2} = 0.707107$   
*shear:* 0  
*circumradius:* 1  
*other names:* tetragonal cuboctahedral wedge  
*comments:* kind of bidiminshed octahedral-cupola (cuboctahedron as "cuboctahedron - 2 squares" and square as "octahedron - 2 square pyramids")  
*cells:* 4+8 square pyramids + 2 cubes + 1 cuboctahedron

#### **4.29 octahedron || cuboctahedron**

*height:*  $1/\sqrt{2} = 0.707107$   
*circumradius:* 1  
*other names:* octahedral cupola, icositetrachoral octahedron-cup, half of icositetrachoron  
*cells:* 1+8 octahedra + 6 square pyramids + 1 cuboctahedron

#### **4.30 octahedron || trigonal cupola**

*height:*  $1/\sqrt{2} = 0.707107$   
*circumradius:* 1  
*other names:* -  
*comments:* kind of diminished octahedral cupola (octahedron as "octahedron - trigon" and trigonal cupola as "cuboctahedron - trigonal cupola")  
*cells:* 2+3 octahedra + 6 square pyramids + 2 trigonal cupolae

#### **4.31 square pyramid || cuboctahedron**

*height:*  $1/\sqrt{2} = 0.707107$   
*circumradius:* 1  
*other names:* -  
*comments:* kind of diminished octahedral cupola (square pyramid as "octahedron - square pyramid" and cuboctahedron as "cuboctahedron - square")  
*cells:* 4 octahedra + 1+1+4+4 square pyramids + 1 cube + 1 cuboctahedron

#### **4.32 square pyramid || trigonal cupola**

*height:*  $1/\sqrt{2} = 0.707107$   
*circumradius:* 1  
*other names:* -  
*comments:* kind of bidiminshed octahedral cupola (square pyramid as "octahedron - square pyramid - trigon" and trigonal cupola as "cuboctahedron - trigon - trigonal cupola")  
*cells:* 1 octahedron + 2+2+4 square pyramids + 1 cube + 2 trigonal cupolae

#### **4.33 trigon || tridiminshed icosahedron**

*height:*  $1/2$   
*shear:*  $(3-\sqrt{5})/(4*\sqrt{3}) = 0.110264$   
*circumradius:* 1  
*other names:* trigonal tridiminshed-icosahedral wedge  
*cells:* 3 tetrahedra + 1 octahedron + 3 square pyramids + 1 trigonal prism + 3 pentagonal pyramids + 1 tridiminshed icosahedron

#### **4.34 pentagon || pentagonal prism**

*height:*  $\sqrt{3/4} = 0.866025$   
*shear:* 0  
*circumradius:*  $\sqrt{(25+3*\sqrt{5})/30} = 1.028076$   
*other names:* trigon-pentagon-diprism, direct sum of trigon and pentagon, pentagonal pentagonal-prismatic wedge  
*comments:* uniform  
*cells:* 5 trigonal prisms + 3 pentagonal prisms

#### **4.35 cube || cuboctahedron**

*height:*  $\sqrt{\sqrt{2}-3/4} = 0.814993$   
*circumradius:*  $\sqrt{(16+6*\sqrt{2})/23} = 1.031784$   
*other names:* cubic cupola  
*cells:* 8 tetrahedra + 6 square antiprisms + 1 cube + 1 cuboctahedron

#### **4.36 icosahedron || icosahedron**

*height:* 1  
*circumradius:*  $\sqrt{(7+\sqrt{5})/8} = 1.074481$   
*other names:* icosahedral prism  
*comments:* uniform  
*cells:* 20 trigonal prisms + 2 icosahedra

#### **4.37 gyroelongated pentagonal pyramid || gyroelongated pentagonal pyramid**

*height:* 1  
*circumradius:*  $\sqrt{(7+\sqrt{5})/8} = 1.074481$   
*other names:* gyroelongated-pentagonal-pyramidal prism  
*comments:* kind of diminished icosahedral-prism (twice: gyroelongated pentagonal pyramid (J11) as "icosahedron - pentagonal pyramid")  
*cells:* 5+5+5 trigonal prisms + 1 pentagonal prism + 2 gyroelongated pentagonal pyramid

#### **4.38 pentagonal pyramid || pentagonal pyramid**

*height:* 1

#### 4.38.1 line || pentagonal prism

*height:*  $\sqrt{(5-\sqrt{5})/10} = 0.525731$   
*circumradius:*  $\sqrt{(7+\sqrt{5})/8} = 1.074481$   
*other names:* pentagonal-pyramidal prism  
*comments:* kind of diminished icosahedral-prism (twice: pentagonal pyramid as "icosahedron - gyroelongated pentagonal pyramid (J11)")  
*cells:* 5 trigonal prisms + 2 pentagonal pyramids + 1 pentagonal prism

#### 4.39 pentagonal antiprism || pentagonal antiprism

*height:* 1

#### 4.39.1 pentagonal prism || gyrated pentagonal prism

*height:*  $\sqrt{(5+\sqrt{5})/10} = 0.850651$   
*circumradius:*  $\sqrt{(7+\sqrt{5})/8} = 1.074481$   
*other names:* pentagonal-antiprismatic prism  
*comments:* uniform, kind of parabidiminished icosahedral-prism (twice: pentagonal antiprism as "icosahedron - 2 pentagonal pyramids")  
*cells:* 10 trigonal prisms + 2 pentagonal antiprisms + 2 pentagonal prisms

#### 4.40 metabidiminished icosahedron || metabidiminished icosahedron

*height:* 1  
*circumradius:*  $\sqrt{(7+\sqrt{5})/8} = 1.074481$   
*other names:* metabidiminished-icosahedral prism  
*comments:* kind of bidiminished icosahedral-prism (twice: metabidiminished icosahedron (J62) as "icosahedron - 2 pentagonal pyramids")  
*cells:* 2+2+2+4 trigonal prisms + 2 pentagonal prisms + 2 metabidiminished icosahedra

#### 4.41 tridiminished icosahedron || tridiminished icosahedron

*height:* 1  
*circumradius:*  $\sqrt{(7+\sqrt{5})/8} = 1.074481$   
*other names:* tridiminished-icosahedral prism  
*comments:* kind of tridiminished icosahedral-prism (twice: tridiminished icosahedron (J63) as "icosahedron - 3 pentagonal pyramids")  
*cells:* 1+1+3 trigonal prisms + 3 pentagonal prisms + 2 tridiminished icosahedra

#### 4.42 pentagonal prism || pentagonal prism

*height:* 1  
*circumradius:*  $\sqrt{(10+\sqrt{5})/10} = 1.106168$   
*other names:* pentagonal-prismatic prism, square-pentagon-diprism, direct sum of square and pentagon

*comments:* uniform  
*cells:* 5 cubes + 4 pentagonal prisms

#### 4.43 cuboctahedron || cuboctahedron

*height:* 1  
*circumradius:*  $\sqrt{5}/2 = 1.118034$   
*other names:* cuboctahedral prism  
*comments:* uniform  
*cells:* 8 trigonal prisms + 6 cubes + 2 cuboctahedra

#### 4.44 trigonal orthobicupola || trigonal orthobicupola

*height:* 1  
*circumradius:*  $\sqrt{5}/2 = 1.118034$   
*other names:* trigonal-orthobicupolaic prism  
*comments:* kind of gyrated cuboctahedral-prism (as 2 trigonal-cupolaic prisms (see 4.45) joined at the hexagonal prism)  
*cells:* 2+6 trigonal prisms + 6 cubes + 2 trigonal orthobicupolae

#### 4.45 trigonal cupola || trigonal cupola

*height:* 1

#### 4.45.1 trigonal prism || hexagonal prism

*height:*  $\sqrt{2/3} = 0.816497$   
*circumradius:*  $\sqrt{5}/2 = 1.118034$   
*other names:* trigonal-cupolaic prism, half of cuboctahedral prism  
*cells:* 1+3 trigonal prisms + 3 cubes + 2 trigonal cupolae + 1 hexagonal prism

#### 4.46 hexagon || hexagonal antiprism

*height:*  $\sqrt{(7-\sqrt{3})/8} = 0.811476$   
*shear:*  $\sqrt{(\sqrt{27}-5)/8} = 0.156586$

#### 4.46.1 hexagon || gyrated hexagonal prism

*height:*  $\sqrt{(\sqrt{3}-5/4)} = 0.694299$   
*shear:* 0  
*circumradius:*  $\sqrt{(19+6*\sqrt{3})/23} = 1.130454$   
*other names:* hexagonal hexagonal-antiprismatic wedge  
*cells:* 6 tetrahedra + 6 square pyramids + 2 hexagonal antiprisms + 1 hexagonal prism

#### 4.47 hexagon || hexagonal prism

*height:*  $\sqrt{3/4} = 0.866025$   
*shear:* 0  
*circumradius:*  $\sqrt{4/3} = 1.154701$   
*other names:* trigon-hexagon-diprism, direct sum of trigon and hexagon, hexagonal hexagonal-

prismatic wedge  
*comments:* uniform  
*cells:* 6 trigonal prisms + 3 hexagonal prisms

#### 4.48 cuboctahedron || truncated tetrahedron

*height:*  $\sqrt{5/8} = 0.790569$   
*circumradius:*  $\sqrt{7/5} = 1.183216$   
*other names:* cuboctahedral monostratic cup of cantellated pentachoron, cuboctahedral monostratic cup of small rhombated pentachoron  
*comments:* kind of diminished cantellated pentachoron (as "cantellated pentachoron - octahedral monostratic cup of cantellated pentachoron (see 4.52)")  
*cells:* 4 octahedra + 6 trigonal prisms + 1 cuboctahedron + 4 trigonal cupolae + 1 truncated tetrahedron

#### 4.49 trigonal orthobicupola || truncated tetrahedron

*height:*  $\sqrt{5/8} = 0.790569$   
*circumradius:*  $\sqrt{7/5} = 1.183216$   
*other names:* -  
*comments:* kind of gyrated cuboctahedral-monostratic-cup-of-cantellated-pentachoron (trigonal orthobicupola (J27) as "2 trigonal cupolae" and truncated tetrahedron as "(truncated tetrahedron - hexahedron) + hexahedron" (see 4.50, 4.51) joined at the hexagonal prism)  
*cells:* 1 octahedron + 3+3 square pyramids + 3+3 trigonal prisms + 1 trigonal orthobicupola + 1+3 trigonal cupolae + 1 truncated tetrahedron

#### 4.50 trigonal cupola || truncated tetrahedron

*height:*  $\sqrt{5/8} = 0.790569$   
*circumradius:*  $\sqrt{7/5} = 1.183216$   
*other names:* -  
*comments:* kind of diminished cuboctahedral-monostratic-cup-of-cantellated-pentachoron (trigonal cupola as "cuboctahedron - trigonal cupola" and truncated tetrahedron as "truncated tetrahedron - hexagon")  
*cells:* 1 octahedron + 3 square pyramids + 3 trigonal prisms + 1+3 trigonal cupolae + 1 hexagonal prism + 1 truncated tetrahedron

#### 4.51 hexagon || trigonal cupola

*height:*  $\sqrt{5/8} = 0.790569$   
*shear:*  $\sqrt{3/8} = 0.612372$

##### 4.51.1 trigon || hexagonal prism

*height:*  $\sqrt{5/12} = 0.645497$   
*shear:* 0  
*circumradius:*  $\sqrt{7/5} = 1.183216$   
*other names:* hexagonal trigonal-cupolaic

wedge  
*comments:* kind of diminished cuboctahedral-monostratic-cup-of-cantellated-pentachoron (trigonal cupola as "cuboctahedron - trigonal cupola" and hexagon as "truncated tetrahedron - truncated tetrahedron")  
*cells:* 3 square pyramids + 3 trigonal prisms + 2 trigonal cupolae + 1 hexagonal prism

#### 4.52 octahedron || truncated tetrahedron

*height:*  $\sqrt{5/8} = 0.790569$   
*circumradius:*  $\sqrt{7/5} = 1.183216$   
*other names:* octahedral monostratic cup of cantellated pentachoron, octahedral monostratic cup of small rhombated pentachoron  
*comments:* kind of diminished cantellated-pentachoron (as "cantellated pentachoron - cuboctahedral monostratic cup of cantellated pentachoron (see 4.48)")  
*cells:* 1 octahedron + 4 trigonal prisms + 4 trigonal cupolae + 1 truncated tetrahedron

#### 4.53 hexagonal antiprism || hexagonal antiprism

*height:* 1

##### 4.53.1 hexagonal prism || gyrated hexagonal prism

*height:*  $\sqrt{\sqrt{3}-1} = 0.855600$   
*circumradius:*  $\sqrt{(4+\sqrt{3})/4} = 1.197085$   
*other names:* hexagonal-antiprismatic prism  
*comments:* uniform  
*cells:* 12 trigonal prisms + 2 hexagonal antiprisms + 2 hexagonal prisms

#### 4.54 hexagonal prism || hexagonal prism

*height:* 1  
*circumradius:*  $\sqrt{3/2} = 1.224745$   
*other names:* hexagonal-prismatic prism, square-hexagon-diprism, direct sum of square and hexagon  
*comments:* uniform  
*cells:* 6 cubes + 4 hexagonal prisms

#### 4.55 truncated tetrahedron || inverse truncated tetrahedron

*height:*  $\sqrt{1/2} = 0.707107$   
*circumradius:*  $\sqrt{3/2} = 1.224745$   
*other names:* equatorial tetrahedral segment of rectified tesseract  
*comments:* weakly uniform  
*cells:* 6 tetrahedra + 8 trigonal cupola + 2 truncated tetrahedra

#### 4.56 tetrahedron || truncated tetrahedron

*height:*  $\sqrt{1/2} = 0.707107$   
*circumradius:*  $\sqrt{3/2} = 1.224745$   
*other names:* tetrahedral monostratic cup of rectified tesseract  
*cells:* 1+4 tetrahedra + 4 trigonal cupolae + 1 truncated tetrahedron

#### 4.57 truncated tetrahedron || truncated tetrahedron

*height:* 1  
*circumradius:*  $\sqrt{13/8} = 1.274755$   
*other names:* truncated-tetrahedral prism  
*comments:* uniform  
*cells:* 4 trigonal prisms + 4 hexagonal prisms + 2 truncated tetrahedra

#### 4.58 octagon || octagonal antiprism

*height:*  $\frac{\sqrt{((2+3*\sqrt{2+\sqrt{2}})/(4+4*\sqrt{2+\sqrt{2}}))^2 + 1}}}{2} = 0.813764$   
*shear:*  $1/\sqrt{(16+4*\sqrt{2}+12*\sqrt{2+\sqrt{2}})} = 0.151048$

##### 4.58.1 octagon || gyrated octagonal prism

*height:*  $\frac{\sqrt{((2+3*\sqrt{2+\sqrt{2}})/(8+4*\sqrt{2+\sqrt{2}}))^2 + 1}}}{2} = 0.700077$   
*shear:* 0  
*circumradius:*  $\frac{\sqrt{((2*\sqrt{2+\sqrt{2}})-\sqrt{2})/(4*\sqrt{2+\sqrt{2}}-3*\sqrt{2}-2))}}{2} = 1.409438$   
*other names:* octagonal octagonal-antiprismatic wedge  
*cells:* 8 tetrahedra + 8 square pyramids + 2 octagonal antiprisms + 1 octagonal prism

#### 4.59 octagon || octagonal prism

*height:*  $\sqrt{3/4} = 0.866025$   
*shear:* 0  
*circumradius:*  $\sqrt{((8+3*\sqrt{2})/6)} = 1.428440$   
*other names:* trigon-octagon-diprism, direct sum of trigon and octagon, octagonal octagonal-prismatic wedge  
*comments:* uniform  
*cells:* 8 trigonal prisms + 3 octagonal prisms

#### 4.60 snub cube || snub cube

*height:* 1  
*circumradius:*  $\frac{\sqrt{((7-8*\cos^2(x))/(12-16*\cos^2(x)))}}{2} = 1.433724$   
*other names:* snub-cubic prism  
*comments:* uniform, x is half of the centriangle underneath an edge of length 1 in the vertex figure of the snub cube:  $\cos(x) =$

$(\text{cbrt}(1+\sqrt{11/27}))+\text{cbrt}(1-\sqrt{11/27}))/\text{cbrt}(\sqrt{128}) = 0.842509$   
*cells:* 8+24 trigonal prisms + 6 cubes + 2 snub cubes

#### 4.61 cuboctahedron || rhombicuboctahedron

*height:*  $\sqrt{((\sqrt{8}-1)/4)} = 0.676097$   
*circumradius:*  $(1+\sqrt{8})/\sqrt{7} = 1.447009$   
*other names:* cuboctahedral cupola  
*cells:* 8 octahedra + 12 square pyramids + 6 square antiprisms + 1 cuboctahedron + 1 rhombicuboctahedron

#### 4.62 cuboctahedron || elongated square cupola

*height:*  $\sqrt{((\sqrt{8}-1)/4)} = 0.676097$   
*circumradius:*  $(1+\sqrt{8})/\sqrt{7} = 1.447009$   
*other names:* -  
*comments:* kind of diminished cuboctahedral-cupola (cuboctahedron as "cuboctahedron - square" and elongated square cupola (J19) as "rhombicuboctahedron - square cupola")  
*cells:* 4 octahedra + 4+4+4 square pyramids + 1+4 square antiprisms + 1 cuboctahedron + 1 elongated square cupola + 1 square cupola

#### 4.63 cuboctahedron || octagonal prism

*height:*  $\sqrt{((\sqrt{8}-1)/4)} = 0.676097$   
*circumradius:*  $(1+\sqrt{8})/\sqrt{7} = 1.447009$   
*other names:* -  
*comments:* kind of bidiminished cuboctahedral-cupola (cuboctahedron as "cuboctahedron - 2 squares" and octagonal prism as "rhombicuboctahedron - 2 square cupolae")  
*cells:* 4+8 square pyramids + 4 square antiprisms + 1 cuboctahedron + 2 square cupolae + 1 octagonal prism

#### 4.64 square || gyrated square cupola

*height:*  $\sqrt{((\sqrt{8}-1)/4)} = 0.676097$   
*shear:*  $1/\sqrt{2} = 0.707107$

##### 4.64.1 octagon || square antiprism

*height:*  $\sqrt{((4-\sqrt{2})/8)} = 0.568527$   
*shear:* 0  
*circumradius:*  $(1+\sqrt{8})/\sqrt{7} = 1.447009$   
*other names:* -  
*comments:* kind of diminished cuboctahedral-cupola (square as "cuboctahedron - cuboctahedron" and square cupola as "rhombicuboctahedron - elongated square cupola")  
*cells:* 8 square pyramids + 1 square antiprism + 2 square cupolae



#### 4.65 octagonal antiprism || octagonal antiprism

height: 1

##### 4.65.1 octagonal prism || gyrated octagonal prism

height:

$$\frac{\sqrt{((1+\sqrt{2+\sqrt{2}})/(2+\sqrt{2+\sqrt{2}}))}}{2} = 0.860296$$

circumradius:  $\frac{\sqrt{((5-2*\sqrt{2+\sqrt{2}})/(8-4*\sqrt{2+\sqrt{2}}))}}{2} = 1.463603$

other names: octagonal-antiprismatic prism

comments: uniform

cells: 16 trigonal prisms + 2 octagonal antiprisms + 2 octagonal prisms

#### 4.66 rhombicuboctahedron || rhombicuboctahedron

height: 1

circumradius:  $\frac{\sqrt{((3+\sqrt{2}))/2)} = 1.485634$

other names: rhombicuboctahedral prism, equatorial monostratic segment of runcinated tesseract, equatorial monostratic segment of runcinated octachoron, equatorial monostratic segment of runcinated hexadecachoron, equatorial monostratic segment of small diprismatotesseractihexadecachoron

comments: uniform, kind of parabidiminished runcinated-tesseract (as "runcinated tesseract - 2 cubic monostratic cups of runcinated tesseract (see 4.71)")

cells: 8 trigonal prisms + 6+12 cubes + 2 rhombicuboctahedra

#### 4.67 elongated square gyrobicupola || elongated square gyrobicupola

height: 1

circumradius:  $\frac{\sqrt{((3+\sqrt{2}))/2)} = 1.485634$

other names: elongated-square-gyrobicupolaic prism

comments: kind of gyrated rhombicuboctahedral-prism (twice: elongated square gyrobicupola (J37) as "elongated square cupola (J19) + square cupola" (see 4.68, 4.69) joined at the octagonal prism)

cells: 8 trigonal prisms + 2+8+8 cubes + 2 elongated square gyrobicupolae

#### 4.68 elongated square cupola || elongated square cupola

height: 1

circumradius:  $\frac{\sqrt{((3+\sqrt{2}))/2)} = 1.485634$

other names: elongated-square-cupolaic prism

comments: kind of diminished rhombicuboctahedral-prism (twice: elongated square cupola (J19) as "rhombicuboctahedron - square cupola")

cells: 4 trigonal prisms + 1+4+4+4 cubes + 2 elongated square cupolae + 1 octagonal prism

#### 4.69 square cupola || square cupola

height: 1

##### 4.69.1 cube || octagonal prism

height:  $1/\sqrt{2} = 0.707107$

circumradius:  $\frac{\sqrt{((3+\sqrt{2}))/2)} = 1.485634$

other names: square-cupolaic prism

comments: kind of bidiminished cubic-monostratic-cup-of-small-diprismatotesseractihexadecachoron (cube as "cube - 2 squares" and octagonal prism as "rhombicuboctahedron - 2 square cupolae") -resp.- kind of diminished rhombicuboctahedral-prism (twice: square cupola as "rhombicuboctahedron - elongated square cupola (J19)")

cells: 4 trigonal prisms + 1+4 cubes + 2 square cupolae + 1 octagonal prism

#### 4.70 octagonal prism || octagonal prism

height: 1

circumradius:  $\frac{\sqrt{((3+\sqrt{2}))/2)} = 1.485634$

other names: octagonal-prismatic prism

comments: uniform, kind of parabidiminished rhombicuboctahedral-prism (twice: octagonal prism as "rhombicuboctahedron - 2 square cupolae")

cells: 8 cubes + 4 octagonal prisms

#### 4.71 cube || rhombicuboctahedron

height:  $1/\sqrt{2} = 0.707107$

circumradius:  $\frac{\sqrt{((3+\sqrt{2}))/2)} = 1.485634$

other names: cubic monostratic cup of runcinated tesseract, cubic monostratic cup of runcinated octachoron, cubic monostratic cup of runcinated hexadecachoron, cubic monostratic cup of small diprismatotesseractihexadecachoron

cells: 8 tetrahedra + 12 trigonal prisms + 1+6 cubes + 1 rhombicuboctahedron

#### 4.72 cube || elongated square cupola

height:  $1/\sqrt{2} = 0.707107$

circumradius:  $\frac{\sqrt{((3+\sqrt{2}))/2)} = 1.485634$

other names: -

comments: kind of diminished cubic-monostratic-cup-of-runcinated-tesseract (cube as "cube - square" and elongated square cupola (J19) as "rhombicuboctahedron - square cupola")

cells: 4 tetrahedra + 4+4 trigonal prisms + 1+1+4 cubes + 1 elongated square cupola + 1 square cupola

#### 4.73 square || square cupola

*height:*  $1/\sqrt{2} = 0.707107$   
*shear:*  $1/2$

##### 4.73.1 octagon || cube

*height:*  $1/2$   
*shear:*  $0$   
*circumradius:*  $\sqrt{((3+\sqrt{2}))/2} = 1.485634$   
*other names:* tetragonal square-cupolaic wedge  
*comments:* kind of diminished cubic-monostratic-cup-of-small-diprismatotesseractihexadecachoron (square as "cube - cube" and square cupola as "rhombicuboctahedron - elongated square cupola")  
*cells:* 4 tetrahedra + 2 square pyramids + 4 trigonal prisms + 1 cube

#### 4.74 dodecahedron || dodecahedron

*height:*  $1$   
*circumradius:*  $\sqrt{((11+3*\sqrt{5}))/8} = 1.487792$   
*other names:* dodecahedral prism  
*comments:* uniform  
*cells:* 12 pentagonal prisms + 2 dodecahedra

#### 4.75 rhombicuboctahedron || truncated octahedron

*height:*  $\sqrt{(\sqrt{2}-3/4)} = 0.814993$   
*circumradius:*  $\sqrt{((35+16*\sqrt{2}))/23} = 1.582890$   
*other names:* -  
*cells:* 12 trigonal prisms + 6 square antiprisms + 8 trigonal cupolae + 1 rhombicuboctahedron + 1 truncated octahedron

#### 4.76 truncated tetrahedron || truncated octahedron

*height:*  $\sqrt{5/8} = 0.790569$   
*circumradius:*  $\sqrt{13/5} = 1.612452$   
*other names:* truncated-tetrahedral monostratic cup of runcinated pentachoron, truncated-tetrahedral monostratic cup of prismatorhombated pentachoron  
*cells:* 6 trigonal prisms + 4 trigonal cupolae + 4 hexagonal prisms + 1 truncated tetrahedron + 1 truncated octahedron

#### 4.77 dodecahedron || icosidodecahedron

*height:*  $(1+\sqrt{5})/4 = 0.809017$   
*circumradius:*  $(1+\sqrt{5})/2 = 1.618034$   
*other names:* dodecahedral cupola  
*cells:* 20 tetrahedra + 12 pentagonal antiprisms + 1 dodecahedron + 1 icosidodecahedron

#### 4.78 icosahedron || dodecahedron

*height:*  $1/2$

*circumradius:*  $(1+\sqrt{5})/2 = 1.618034$   
*other names:* icosahedral antiprism, dodecahedral antiprism  
*cells:* 20+30 tetrahedra + 1 icosahedron + 12 pentagonal pyramids + 1 dodecahedron

#### 4.79 gyroelongated pentagonal pyramid || dodecahedron

*height:*  $1/2$   
*circumradius:*  $(1+\sqrt{5})/2 = 1.618034$   
*other names:* -  
*comments:* kind of diminished dodecahedral-antiprism (gyroelongated pentagonal pyramid (J11) as "icosahedron - pentagonal pyramid" and dodecahedron as "dodecahedron - pentagon")  
*cells:* 5+5+5+5+5+5+10 tetrahedra + 1 gyroelongated pentagonal pyramid + 1+5+5 pentagonal pyramids + 1 pentagonal antiprism + 1 dodecahedron

#### 4.80 pentagon || gyrated pentagonal pyramid

*height:*  $1/2$   
*shear:*  $\sqrt{((25+11*\sqrt{5}))/40} = 1.113516$

##### 4.80.1 point || pentagonal antiprism

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*shear:*  $0$   
*circumradius:*  $(1+\sqrt{5})/2 = 1.618034$   
*other names:* pentagonal-antiprismatic pyramid  
*comments:* parabidiminished icosahedral pyramid, kind of diminished dodecahedral-antiprism (pentagonal pyramid as "icosahedron - gyroelongated pentagonal pyramid (J11)" and pentagon as "dodecahedron - dodecahedron")  
*cells:* 10 tetrahedra + 2 pentagonal pyramids + 1 pentagonal antiprism

#### 4.81 pentagonal antiprism || dodecahedron

*height:*  $1/2$   
*circumradius:*  $(1+\sqrt{5})/2 = 1.618034$   
*other names:* -  
*comments:* pentagonal-antiprismatic monostratic cup of great antiprism, kind of bidiminished dodecahedral-antiprism (pentagonal antiprism as "icosahedron - 2 pentagonal pyramids" and dodecahedron " as dodecahedron - 2 pentagons")  
*cells:* 10+10+10 tetrahedra + 10 pentagonal pyramids + 1+2 pentagonal antiprisms + 1 dodecahedron

#### 4.82 metabidiminished icosahedron || dodecahedron

*height:*  $1/2$   
*circumradius:*  $(1+\sqrt{5})/2 = 1.618034$

*other names:* -  
*comments:* kind of bidiminished dodecahedral-antiprism (metabidiminished icosahedron (J62) as "icosahedron - 2 pentagonal pyramids" and dodecahedron " as dodecahedron - 2 pentagons")  
*cells:* 1+1+2+2+2+2+4+4+4+4+4 tetrahedra + 2+2+2+4 pentagonal pyramids + 2 pentagonal antiprisms + 1 metabidiminished icosahedron + 1 dodecahedron

#### 4.83 tridiminished icosahedron || dodecahedron

*height:* 1/2  
*circumradius:*  $(1+\sqrt{5})/2 = 1.618034$   
*other names:* -  
*comments:* kind of tridiminished dodecahedral-antiprism (tridiminished icosahedron (J63) as "icosahedron - 3 pentagonal pyramids" and dodecahedron " as dodecahedron - 3 pentagons")  
*cells:* 1+1+3+3+3+3+6 tetrahedra + 3+3+3 pentagonal pyramids + 3 pentagonal antiprisms + 1 tridiminished icosahedron + 1 dodecahedron

#### 4.84 point || icosahedron

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*shear:* 0  
*circumradius:*  $(1+\sqrt{5})/2 = 1.618034$   
*other names:* icosahedral pyramid  
*comments:* homohedral  
*cells:* 20 tetrahedra + 1 icosahedron

#### 4.85 point || gyroelongated pentagonal pyramid

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*shear:* 0  
*circumradius:*  $(1+\sqrt{5})/2 = 1.618034$   
*other names:* gyroelongated-pentagonal-pyramidal pyramid  
*comments:* kind of diminished icosahedral-pyramid (point as "point - point" and gyroelongated pentagonal pyramid (J11) as "icosahedron - pentagonal pyramid")  
*cells:* 5+5+5 tetrahedra + 1 gyroelongated pentagonal pyramid + 1 pentagonal pyramid

#### 4.86 point || pentagonal pyramid

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*shear:* 0

#### 4.86.1 line || perpendicular pentagon

*height:*  $\sqrt{(5-2*\sqrt{5})/20} = 0.162460$   
*shear (top):* 0  
*shear (bottom):* 0  
*circumradius:*  $(1+\sqrt{5})/2 = 1.618034$   
*other names:* pentagonal-pyramidal pyramid  
*comments:* selfdual, kind of diminished

icosahedral-pyramid (point as "point - point" and pentagonal pyramid as "icosahedron - gyroelongated pentagonal pyramid (J11)")  
*cells:* 5 tetrahedra + 2 pentagonal pyramids

#### 4.87 point || metabidiminished icosahedron

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*shear:* 0  
*circumradius:*  $(1+\sqrt{5})/2 = 1.618034$   
*other names:* metabidiminished-icosahedral pyramid  
*comments:* kind of bidiminished icosahedral-pyramid (point as "point - 2 points" and metabidiminished icosahedron (J62) as "icosahedron - 2 pentagonal pyramids")  
*cells:* 2+2+2+4 tetrahedra + 2 pentagonal pyramids + 1 metabidiminished icosahedron

#### 4.88 point || tridiminished icosahedron

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*shear:* 0  
*circumradius:*  $(1+\sqrt{5})/2 = 1.618034$   
*other names:* tridiminished-icosahedral pyramid  
*comments:* kind of tridiminished icosahedral-pyramid (point as "point - 3 points" and metabidiminished icosahedron (J63) as "icosahedron - 3 pentagonal pyramids")  
*cells:* 1+1+3 tetrahedra + 3 pentagonal pyramids + tridiminished icosahedron

#### 4.89 truncated octahedron || truncated octahedron

*height:* 1  
*circumradius:*  $\sqrt{11/4} = 1.658312$   
*other names:* truncated-octahedral prism  
*comments:* uniform  
*cells:* 6 cubes + 8 hexahedral prisms + 2 truncated octahedra

#### 4.90 icosidodecahedron || icosidodecahedron

*height:* 1  
*circumradius:*  $\sqrt{(7+2*\sqrt{5})/2} = 1.693527$   
*other names:* icosidodecahedral prism  
*comments:* uniform  
*cells:* 20 trigonal prisms + 12 pentagonal prisms + 2 icosidodecahedra

#### 4.91 orthobirotunda || orthobirotunda

*height:* 1  
*circumradius:*  $\sqrt{(7+2*\sqrt{5})/2} = 1.693527$   
*other names:* orthobirotundaic prism

*comments:* kind of gyrated icosidodecahedral-prism (twice: orthobirotunda as "rotunda + rotunda" (see 4.92) joined at the decagonal prism)  
*cells:* 10+10 trigonal prisms + 2+10 pentagonal prisms + 2 orthobirotundae

#### 4.92 rotunda || rotunda

*height:* 1  
*circumradius:*  $\sqrt{7+2\sqrt{5}}/2 = 1.693527$   
*other names:* rotundaic prism, half of icosidodecahedral prism  
*cells:* 5+5 trigonal prisms + 1+5 pentagonal prisms + 2 rotundae + 1 decagonal prism

#### 4.93 decagon || decagonal antiprism

*height:*  $\frac{\sqrt{(\sqrt{8}+3\sqrt{5+\sqrt{5}})/(2\sqrt{8}+4\sqrt{5+\sqrt{5}}))}}{2} = 0.814774$   
*shear:*  $1/\sqrt{(18+2\sqrt{5}+3\sqrt{8}\sqrt{5+\sqrt{5}})} = 0.148581$

##### 4.93.1 decagon || gyrated decagonal prism

*height:*  $\frac{\sqrt{(\sqrt{8}+3\sqrt{5+\sqrt{5}})/(4\sqrt{8}+4\sqrt{5+\sqrt{5}}))}}{2} = 0.702658$   
*shear:* 0  
*circumradius:*  $\frac{\sqrt{(\sqrt{8}\sqrt{5+\sqrt{5}}-1-\sqrt{5})/(2\sqrt{8}\sqrt{5+\sqrt{5}}-7-3\sqrt{5})}}{2} = 1.702385$   
*other names:* decagonal decagonal-antiprismatic wedge  
*cells:* 10 tetrahedra + 10 square pyramids + 2 decagonal antiprisms + 1 decagonal prism

#### 4.94 decagon || decagonal prism

*height:*  $\sqrt{3/4} = 0.866025$   
*shear:* 0  
*circumradius:*  $\sqrt{(11+3\sqrt{5})/6} = 1.717954$   
*other names:* trigon-decagon-diprism, direct sum of trigon and decagon, decagonal decagonal-prismatic wedge  
*comments:* uniform  
*cells:* 10 trigonal prisms + 3 decagonal prisms

#### 4.95 cuboctahedron || truncated octahedron

*height:*  $1/\sqrt{2} = 0.707107$   
*circumradius:*  $\sqrt{3} = 1.732051$   
*other names:* cuboctahedral monostratic cup of rectified icositetrahedron  
*cells:* 6 cube + 1 cuboctahedron + 8 trigonal cupolae + 1 truncated octahedron

#### 4.96 decagonal antiprism || decagonal antiprism

*height:* 1

##### 4.96.1 decagonal prism || gyrated decagonal prism

*height:*  $\frac{\sqrt{(\sqrt{2}+\sqrt{5+\sqrt{5}})/(2\sqrt{2}+\sqrt{5+\sqrt{5}}))}}{2} = 0.862397$   
*circumradius:*  $\frac{\sqrt{(5\sqrt{2}-2\sqrt{5+\sqrt{5}})/(8\sqrt{2}-4\sqrt{5+\sqrt{5}}))}}{2} = 1.747560$   
*other names:* decagonal-antiprismatic prism  
*comments:* uniform  
*cells:* 20 trigonal prisms + 2 decagonal antiprisms + 2 decagonal prisms

#### 4.97 decagonal prism || decagonal prism

*height:* 1  
*circumradius:*  $\sqrt{(4+\sqrt{5})/4} = 1.765796$   
*other names:* decagonal-prismatic prism, square-decagon-diprism, direct sum of square and decagon  
*comments:* uniform  
*cells:* 10 cubes + 4 decagonal prisms

#### 4.98 truncated octahedron || truncated cube

*height:*  $\sqrt{(\sqrt{8}-1)/4} = 0.676097$   
*circumradius:*  $\sqrt{(11+8\sqrt{2})/7} = 1.785406$   
*other names:* -  
*cells:* 12 tetrahedra + 8 trigonal cupolae + 6 square cupolae + 1 truncated octahedron + 1 truncated cube

#### 4.99 truncated cube || truncated cube

*height:* 1  
*circumradius:*  $\sqrt{2+\sqrt{2}} = 1.847759$   
*other names:* truncated-cubic prism, equatorial rhombicuboctahedral segment of small rhombated tesseract, equatorial rhombicuboctahedral segment of cantellated tesseract  
*comments:* uniform  
*cells:* 8 trigonal prisms + 6 octagonal prisms + 2 truncated cubes

#### 4.100 rhombicuboctahedron || truncated cube

*height:*  $1/\sqrt{2} = 0.707107$   
*circumradius:*  $\sqrt{2+\sqrt{2}} = 1.847759$   
*other names:* rhombicuboctahedral monostratic cup of cantellated octachoron, rhombicuboctahedral monostratic cup of cantellated tesseract, rhombicuboctahedral monostratic cup of small rhombated tesseract

*cells:* 8 octahedra + 12 trigonal prisms + 1 rhombicuboctahedron + 6 square cupolae + 1 truncated cube

#### **4.101 elongated square gyrobicupola || truncated cube**

*height:*  $1/\sqrt{2} = 0.707107$   
*circumradius:*  $\sqrt{2+\sqrt{2}} = 1.847759$   
*other names:* -  
*comments:* 2 kinds of gyrated rhombicuboctahedral-monostratic-cup-of-cantellated-octachoron (elongated square gyrobicupola (J37) as "elongated square cupola (J19) + square cupola" (depending on which being gyrated) and truncated cube as "truncated cube + octagon" (see 4.103 resp. 4.104, and 4.105) joined at the octagonal prism)  
*cells:* 4 octahedra + 4+4 square pyramids + 4+4+4 trigonal prisms + 1 elongated square gyrobicupola + 1+1+4 square cupolae + 1 truncated cube

#### **4.102 rhombicuboctahedron || gyrated truncated cube**

*height:*  $1/\sqrt{2} = 0.707107$   
*circumradius:*  $\sqrt{2+\sqrt{2}} = 1.847759$   
*other names:* -  
*comments:* kind of bigyrated rhombicuboctahedral-monostratic-cup-of-cantellated-octachoron (rhombicuboctahedron as "octagonal prism + 2 square cupolae" and truncated cube as "truncated cube + 2 octagons" (see 4.104, 4.105) joined at the octagonal prisms)  
*cells:* 8+8 square pyramids + 4+8 trigonal prisms + 1 rhombicuboctahedron + 2+4 square cupolae + 1 truncated cube

#### **4.103 elongated square cupola || truncated cube**

*height:*  $1/\sqrt{2} = 0.707107$   
*circumradius:*  $\sqrt{2+\sqrt{2}} = 1.847759$   
*other names:* -  
*comments:* kind of diminished rhombicuboctahedral-monostratic-cup-of-cantellated-octachoron (elongated square cupola (J19) as "rhombicuboctahedron - square cupola" and truncated cube as "truncated cube - octagon")  
*cells:* 4 octahedra + 4 square pyramids + 4+4 triangular prisms + 1 elongated square cupola + 1+4 square cupolae + 1 octagonal prism + 1 truncated cube

#### **4.104 elongated square cupola || gyrated truncated cube**

*height:*  $1/\sqrt{2} = 0.707107$   
*circumradius:*  $\sqrt{2+\sqrt{2}} = 1.847759$   
*other names:* -  
*comments:* kind of diminished gyrated rhombicuboctahedral-monostratic-cup-of-

cantellated-octachoron (elongated square cupola (J19) as "rhombicuboctahedron - 2 square cupolae + square cupola" and truncated cube as "truncated cube - 2 octagons + octagon": diminishing 4.102 resp. gyrating 4.103 as "4.106 + 4.105" joining at an octagonal prism)

*cells:* 4+4+4 square pyramids + 4+4 trigonal prisms + 1 elongated square cupola + 1+4 square cupolae + 1 octagonal prism + 1 truncated cube

#### **4.105 octagon || square cupola**

*height:*  $1/\sqrt{2} = 0.707107$   
*shear:*  $(1+\sqrt{2})/2 = 1.207107$

##### **4.105.1 square || octagonal prism**

*height:*  $1/2$   
*shear:* 0  
*circumradius:*  $\sqrt{2+\sqrt{2}} = 1.847759$   
*other names:* -  
*comments:* kind of diminished gyrated rhombicuboctahedral-monostratic-cup-of-cantellated-octachoron (square cupola as "rhombicuboctahedron - elongated square cupola (J19)" and octagon as "truncated cube - truncated cube") -resp.- kind of bidiminished octahedral-monostratic-cup-of-runcinated-icositetrahedron (square as "octahedron - 2 square pyramids" and octagonal prism as "rhombicuboctahedron - 2 square cupolae")

*cells:* 4 square pyramids + 4 trigonal prisms + 2 square cupolae + 1 octagonal prism

#### **4.106 octagonal prism || truncated cube**

*height:*  $1/\sqrt{2} = 0.707107$   
*circumradius:*  $\sqrt{2+\sqrt{2}} = 1.847759$   
*other names:* -  
*comments:* kind of bidiminished gyrated rhombicuboctahedral-monostratic-cup-of-cantellated-octachoron (octagonal prism as "rhombicuboctahedron - 2 square cupolae" and truncated cube as "truncated cube - 2 octagons")  
*cells:* 8 square pyramids + 4 trigonal prisms + 4 square cupolae + 1+2 octagonal prisms + 1 truncated cube

#### **4.107 octahedron || rhombicuboctahedron**

*height:*  $1/2$   
*circumradius:*  $\sqrt{2+\sqrt{2}} = 1.847759$   
*other names:* octahedral monostratic cup of runcinated icositetrahedron, octahedral monostratic cup of small prisma-tetraconta-octachoron  
*cells:* 1 octachoron + 6 square pyramids + 8+12 trigonal prisms + 1 rhombicuboctahedron

#### 4.108 square pyramid || elongated square cupola

*height:* 1/2  
*circumradius:*  $\sqrt{2+\sqrt{2}} = 1.847759$   
*other names:* -  
*comments:* kind of diminished octahedral-monostatic-cup-of-runcinated-icositetrachoron (square pyramid as "octahedron - square pyramid" and elongated square cupola (J19) as "rhombicuboctahedron - square cupola")  
*cells:* 1+1+4 square pyramids + 4+4+4 trigonal prisms + 1 elongated square cupola + 1 square cupola

#### 4.109 square pyramid || square cupola

*height:* 1/2  
*circumradius:*  $\sqrt{2+\sqrt{2}} = 1.847759$   
*other names:* -  
*comments:* kind of diminished octahedral-monostatic-cup-of-runcinated-icositetrachoron (square pyramid as "octahedron - square pyramid" and square cupola as "rhombicuboctahedron - elongated square cupola (J19) ")  
*cells:* 8 square pyramids + 2 trigonal prisms + 2 square cupolae

#### 4.110 snub dodecahedron || snub dodecahedron

*height:* 1  
*circumradius:*  $\sqrt{((7-8*\cos^2(x))/(12-16*\cos^2(x)))} = 2.213060$   
*other names:* snub-dodecahedral prism  
*comments:* uniform, x is half of the centriangle underneath an edge of length 1 in the vertex figure of the snub dodecahedron:  $\cos(x) = (\text{cb}rt(9+9*\sqrt{5}+\sqrt{102+162*\sqrt{5}})+\text{cb}rt(9+9*\sqrt{5}-\sqrt{102+162*\sqrt{5}}))/\text{cb}rt(288) = 0.857781$   
*cells:* 20+60 trigonal prisms + 12 pentagonal prisms + 2 snub dodecahedra

#### 4.111 rhombicosidodecahedron || rhombicosidodecahedron

*height:* 1  
*circumradius:*  $\sqrt{3+\sqrt{5}} = 2.288246$   
*other names:* rhombicosidodecahedral prism  
*comments:* uniform  
*cells:* 20 trigonal prisms + 30 cubes + 12 pentagonal prisms + 2 rhombicosidodecahedra

#### 4.112 gyrated rhombicosidodecahedron || gyrated rhombicosidodecahedron

*height:* 1  
*circumradius:*  $\sqrt{3+\sqrt{5}} = 2.288246$   
*other names:* gyrated-rhombicosidodecahedral

prism  
*comments:* kind of gyrated rhombicosidodecahedral-prism (twice: gyrated rhombicosidodecahedron (J72) as "diminished rhombicosidodecahedron (J76) + pentagonal cupola" (see 4.116, 4.117) joined at the decagonal prism)  
*cells:* 5+5+5+5 trigonal prisms + 5+5+5+5+10 cubes + 1+1+5+5 pentagonal prisms + 2 gyrated rhombicosidodecahedra

#### 4.113 parabigyrate rhombicosidodecahedron || parabigyrate rhombicosidodecahedron

*height:* 1  
*circumradius:*  $\sqrt{3+\sqrt{5}} = 2.288246$   
*other names:* parabigyrate-rhombicosidodecahedral prism  
*comments:* kind of parabigyrate rhombicosidodecahedral-prism (twice: parabigyrate rhombicosidodecahedron (J73) as "parabidiminished rhombicosidodecahedron (J80) + 2 pentagonal cupolae" (see 4.121, 4.117) joined at the decagonal prisms)  
*cells:* 10+10 trigonal prisms + 10+10+10 cubes + 2+10 pentagonal prisms + 2 parabigyrate rhombicosidodecahedra

#### 4.114 metabigyrate rhombicosidodecahedron || metabigyrate rhombicosidodecahedron

*height:* 1  
*circumradius:*  $\sqrt{3+\sqrt{5}} = 2.288246$   
*other names:* metabigyrate-rhombicosidodecahedral prism  
*comments:* kind of metabigyrate rhombicosidodecahedral-prism (twice: metabigyrate rhombicosidodecahedron (J74) as "metabidiminished rhombicosidodecahedron (J81) + 2 pentagonal cupolae" (see 4.122, 4.117) joined at the decagonal prisms)  
*cells:* 2+2+2+2+4+4+4 trigonal prisms + 1+1+2+2+4+4+4+4+4+4 cubes + 2+2+2+2+4 pentagonal prisms + 2 metabigyrate rhombicosidodecahedra

#### 4.115 trigyrate rhombicosidodecahedron || trigyrate rhombicosidodecahedron

*height:* 1  
*circumradius:*  $\sqrt{3+\sqrt{5}} = 2.288246$   
*other names:* trigyrate-rhombicosidodecahedral prism  
*comments:* kind of trigyrate rhombicosidodecahedral-prism (twice: trigyrate

rhombicosidodecahedron (J75) as "tridiminished rhombicosidodecahedron (J83) + 3 pentagonal cupolae" (see xxx, 4.117) joined at the decagonal prisms)

*cells:* 1+1+3+3+6+6 trigonal prisms + 3+3+3+3+6+6+6 cubes + 3+3+3+3 pentagonal prisms + 2 trigyrated rhombicosidodecahedra

#### **4.116 diminished rhombicosidodecahedron || diminished rhombicosidodecahedron**

*height:* 1  
*circumradius:*  $\sqrt{3+\sqrt{5}} = 2.288246$   
*other names:* diminished-rhombicosidodecahedral prism  
*comments:* kind of diminished rhombicosidodecahedral-prism (twice: diminished rhombicosidodecahedron (J76) as "rhombicosidodecahedron - pentagonal cupola")  
*cells:* 5+5+5 trigonal prisms + 5+5+5+10 cubes + 1+5+5 pentagonal prisms + 1 decagonal prism + 2 diminished rhombicosidodecahedra

#### **4.117 pentagonal cupola || pentagonal cupola**

*height:* 1

##### **4.117.1 pentagonal prism || decagonal prism**

*height:*  $\sqrt{(5-\sqrt{5})/10} = 0.525731$   
*circumradius:*  $\sqrt{3+\sqrt{5}} = 2.288246$   
*other names:* pentagonal-cupolaic prism  
*comments:* kind of diminished rhombicosidodecahedral-prism (twice: pentagonal cupola as "rhombicosidodecahedron - diminished rhombicosidodecahedron (J76)")  
*cells:* 5 trigonal prisms + 5 cubes + 1 pentagonal prism + 1 decagonal prism + 2 pentagonal cupolae

#### **4.118 gyrated paradiminished rhombicosidodecahedron || gyrated paradiminished rhombicosidodecahedron**

*height:* 1  
*circumradius:*  $\sqrt{3+\sqrt{5}} = 2.288246$   
*other names:* gyrated-paradiminished-rhombicosidodecahedral prism  
*comments:* kind of diminished gyrated-rhombicosidodecahedral-prism (twice: gyrated paradiminished rhombicosidodecahedron (J77) as "gyrated rhombicosidodecahedron (J72) - pentagonal cupola") -resp.- kind of gyrated diminished-rhombicosidodecahedral-prism (twice: gyrated paradiminished rhombicosidodecahedron (J77) as "paradiminished

rhombicosidodecahedron (J80) + pentagonal cupola" (see 4.121, 4.117) joined at the decagonal prisms)

*cells:* 5+5+5 trigonal prisms + 5+5+5+10 cubes + 1+5+5 pentagonal prisms + 1 decagonal prism + 2 gyrated paradiminished rhombicosidodecahedra

#### **4.119 gyrated metadiminished rhombicosidodecahedron || gyrated metadiminished rhombicosidodecahedron**

*height:* 1  
*circumradius:*  $\sqrt{3+\sqrt{5}} = 2.288246$   
*other names:* gyrated-metadiminished-rhombicosidodecahedral prism  
*comments:* kind of diminished gyrated-rhombicosidodecahedral-prism (twice: gyrated metadiminished rhombicosidodecahedron (J78) as "gyrated rhombicosidodecahedron (J72) - pentagonal cupola") -resp.- kind of gyrated diminished-rhombicosidodecahedral-prism (twice: gyrated metadiminished rhombicosidodecahedron (J78) as "metadiminished rhombicosidodecahedron (J81) + pentagonal cupola" (see 4.122, 4.117) joined at the decagonal prisms)  
*cells:* 1+1+1+2+2+2+2+2+2 trigonal prisms + 1+1+1+2+2+2+2+2+2+2+2+2+2+2+2 cubes + 1+1+1+2+2+2+2 pentagonal prisms + 1 decagonal prism + 2 gyrated metadiminished rhombicosidodecahedra

#### **4.120 bigyrated diminished rhombicosidodecahedron || bigyrated diminished rhombicosidodecahedron**

*height:* 1  
*circumradius:*  $\sqrt{3+\sqrt{5}} = 2.288246$   
*other names:* bigyrated-diminished-rhombicosidodecahedral prism  
*comments:* kind of diminished bigyrated-rhombicosidodecahedral-prism (twice: bigyrated diminished rhombicosidodecahedron (J79) as "metabigyrate rhombicosidodecahedron (J74) - pentagonal cupola") -resp.- kind of gyrated gyrate-metadiminished-rhombicosidodecahedral-prism (twice: bigyrate diminished rhombicosidodecahedron (J79) as "gyrate metabidiminished rhombicosidodecahedron (J82) + pentagonal cupola" (see 4.123, 4.117) joined at the decagonal prism) -resp.- kind of bigyrate diminished-rhombicosidodecahedral-prism (twice: bigyrate diminished rhombicosidodecahedron (J79) as "tridiminished rhombicosidodecahedron (J83) + 2 pentagonal cupolae" (see 4.124, 4.117) joined at the decagonal prisms)  
*cells:* 1+1+1+2+2+2+2+2+2+2+2+2+2+2+2 cubes

+ 1+1+1+2+2+2+2 pentagonal prisms + 1 decagonal prism + 2 gyrated bidiminshed rhombicosidodecahedra

**4.121 parabidiminished rhombicosidodecahedron || parabidiminished rhombicosidodecahedron**

*height:* 1  
*circumradius:*  $\sqrt{3+\sqrt{5}} = 2.288246$   
*other names:* parabidiminished-rhombicosidodecahedral prism  
*comments:* kind of parabidiminished rhombicosidodecahedral-prism (twice: parabidiminished rhombicosidodecahedron (J80) as "rhombicosidodecahedron - 2 pentagonal cupolae")  
*cells:* 10 trigonal prisms + 10+10 cubes + 10 pentagonal prisms + 2 decagonal prisms + 2 parabidiminished rhombicosidodecahedra

**4.122 metabidiminished rhombicosidodecahedron || metabidiminished rhombicosidodecahedron**

*height:* 1  
*circumradius:*  $\sqrt{3+\sqrt{5}} = 2.288246$   
*other names:* metabidiminished-rhombicosidodecahedral prism  
*comments:* kind of metabidiminished rhombicosidodecahedral-prism (twice: metabidiminished rhombicosidodecahedron (J81) as "rhombicosidodecahedron - 2 pentagonal cupolae")  
*cells:* 2+2+2+4 trigonal prisms + 1+1+2+4+4+4+4 cubes + 2+2+2+4 pentagonal prisms + 2 decagonal prisms + 2 metabidiminished rhombicosidodecahedra

**4.123 gyrated bidiminished rhombicosidodecahedron || gyrated bidiminished rhombicosidodecahedron**

*height:* 1  
*circumradius:*  $\sqrt{3+\sqrt{5}} = 2.288246$   
*other names:* gyrated-bidiminished-rhombicosidodecahedral prism  
*comments:* kind of bidiminished gyrated-rhombicosidodecahedral-prism (twice: gyrated bidiminished rhombicosidodecahedron (J82) as "gyrated rhombicosidodecahedron (J72) - 2 pentagonal cupolae") -resp.- kind of gyrated metabidiminished-rhombicosidodecahedral-prism (twice: gyrated bidiminished rhombicosidodecahedron (J82) as "tridiminished rhombicosidodecahedron (J83) + pentagonal cupola" (see 4.124, 4.117) joined at the decagonal prism)

*cells:* 1+1+1+1+2+2+2 trigonal prisms + 1+1+1+1+2+2+2+2+2+2+2+2+2+2+2+2 cube + 1+1+1+1+2+2+2 pentagonal prisms + 2 decagonal prisms + 2 gyrated bidiminished rhombicosidodecahedra

**4.124 tridiminished rhombicosidodecahedron || tridiminished rhombicosidodecahedron**

*height:* 1  
*circumradius:*  $\sqrt{3+\sqrt{5}} = 2.288246$   
*other names:* tridiminished-rhombicosidodecahedral prism  
*comments:* kind of tridiminished rhombicosidodecahedral-prism (twice: tridiminished rhombicosidodecahedron (J83) as "rhombicosidodecahedron - 3 pentagonal cupolae")  
*cells:* 1+1+3 trigonal prisms + 3+3+3+6 cubes + 3+3+3 pentagonal prisms + 3 decagonal prisms + 2 tridiminished rhombicosidodecahedra

**4.125 truncated cuboctahedron || truncated cuboctahedron**

*height:* 1  
*circumradius:*  $\sqrt{(7+3*\sqrt{2})/2} = 2.370932$   
*other names:* truncated-cuboctahedral prism, great-rhombicosidodecahedral prism  
*comments:* uniform  
*cells:* 12 cubes + 8 hexagonal prisms + 6 octagonal prisms + 2 truncated cuboctahedra

**4.126 rhombicosidodecahedron || truncated icosahedron**

*height:*  $(1+\sqrt{5})/4 = 0.809017$   
*circumradius:*  $\sqrt{(106+41*\sqrt{5})/32} = 2.485450$   
*other names:* -  
*cells:* 30 trigonal prisms + 12 pentagonal antiprisms + 20 trigonal cupolae + 1 rhombicosidodecahedron + 1 truncated icosahedron

**4.127 truncated icosahedron || truncated icosahedron**

*height:* 1  
*circumradius:*  $\sqrt{(31+9*\sqrt{5})/8} = 2.527959$   
*other names:* truncated-icosahedral prism  
*comments:* uniform  
*cells:* 12 pentagonal prisms + 20 hexagonal prisms + 2 truncated dodecahedra

**4.128 truncated cube || truncated cuboctahedron**

*height:*  $1/\sqrt{2} = 0.707107$   
*circumradius:*  $\sqrt{4+\sqrt{8}} = 2.613126$   
*other names:* truncated-cubical monostratic cup of runcinated tesseract, truncated-cubical monostratic cup of runcinated octachoron,



truncated-cubical monostratic cup of  
 prismatorhombated hexadecachoron  
*cells:* 12 trigonal prisms + 8 trigonal  
 cupolae + 6 octagonal prisms + 1 truncated cube +  
 1 truncated cuboctahedron

#### **4.129 cuboctahedron || truncated cube**

*height:* 1/2  
*circumradius:*  $\sqrt{4+\sqrt{8}} = 2.613126$   
*other names:* cuboctahedral monostratic cup of  
 cantellated icositetrachoron, cuboctahedral  
 monostratic cup of small rhombated  
 icositetrachoron  
*cells:* 8 trigonal prisms + 1  
 cuboctahedron + 6 square cupolae + 1 truncated  
 cube

#### **4.130 truncated dodecahedron || truncated dodecahedron**

*height:* 1  
*circumradius:*  $\sqrt{((39+15*\sqrt{5}))/8} =$   
 3.011250  
*other names:* truncated-dodecahedral prism  
*comments:* uniform  
*cells:* 20 trigonal prisms + 12 decagonal  
 prisms + 2 truncated dodecahedra

#### **4.131 icosidodecahedron || rhombicosidodecahedron**

*height:* 1/2  
*circumradius:*  $\sqrt{5+2*\sqrt{5}} = 3.077684$   
*other names:* icosidodecahedral cupola, second  
 icosahedral monostratic segment of rectified  
 hexacosichoron  
*cells:* 20 octahedra + 30 square  
 pyramids + 12 pentagonal antiprisms + 1  
 icosidodecahedron + 1 rhombicosidodecahedron

#### **4.132 icosidodecahedron || diminished rhombicosidodecahedron**

*height:* 1/2  
*circumradius:*  $\sqrt{5+2*\sqrt{5}} = 3.077684$   
*other names:* -  
*comments:* kind of diminished  
 icosidodecahedral-cupola (icosidodecahedron as  
 "icosidodecahedron - pentagon" and diminished  
 rhombicosidodecahedron (J76) as  
 "rhombicosidodecahedron - pentagonal cupola")  
*cells:* 5+5+5 octahedra + 5+5+5+5+10  
 square pyramids + 1+5+5 pentagonal antiprisms +  
 1 icosidodecahedron + 1 diminished  
 rhombicosidodecahedron + 1 pentagonal cupola

#### **4.133 pentagon || gyrated pentagonal cupola**

*height:* 1/2  
*shear:*  $\sqrt{(5+2*\sqrt{5})/5} = 1.376382$

#### **4.133.1 decagon || pentagonal antiprism**

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*shear:* 0  
*circumradius:*  $\sqrt{5+2*\sqrt{5}} = 3.077684$   
*other names:* -  
*comments:* kind of diminished  
 icosidodecahedral-cupola (pentagon as  
 "icosidodecahedron - icosidodecahedron" and  
 pentagonal cupola as "rhombicosidodecahedron -  
 diminished rhombicosidodecahedron") -resp.- kind  
 of bidiminished icosahedral-cupola (decagon as  
 "icosidodecahedra - 2 rotunda" and pentagonal  
 antiprism as "icosahedron - 2 pentagonal  
 pyramids")  
*cells:* 10 square pyramids + 1  
 pentagonal antiprism + 2 pentagonal cupolae

#### **4.134 icosidodecahedron || parabidiminished rhombicosidodecahedron**

*height:* 1/2  
*circumradius:*  $\sqrt{5+2*\sqrt{5}} = 3.077684$   
*other names:* -  
*comments:* kind of bidiminished  
 icosidodecahedral-cupola (icosidodecahedron as  
 "icosidodecahedron - 2 pentagons" and  
 parabidiminished rhombicosidodecahedron (J80) as  
 "rhombicosidodecahedron - 2 pentagonal cupolae")  
*cells:* 10 octahedra + 10+10+10 square  
 pyramids + 10 pentagonal antiprisms + 1  
 icosidodecahedron + 2 pentagonal cupolae + 1  
 parabidiminished rhombicosidodecahedron

#### **4.135 icosidodecahedron || metabidiminished rhombicosidodecahedron**

*height:* 1/2  
*circumradius:*  $\sqrt{5+2*\sqrt{5}} = 3.077684$   
*other names:* -  
*comments:* kind of bidiminished  
 icosidodecahedral-cupola (icosidodecahedron as  
 "icosidodecahedron - 2 pentagons" and  
 metabidiminished rhombicosidodecahedron (J81)  
 as "rhombicosidodecahedron - 2 pentagonal  
 cupolae")  
*cells:* 2+2+2+4 octahedra +  
 1+1+2+2+4+4+4+4+4+4 square pyramids +  
 2+2+2+4 pentagonal antiprisms + 1  
 icosidodecahedron + 2 pentagonal cupolae + 1  
 metabidiminished rhombicosidodecahedron

#### **4.136 icosidodecahedron || tridiminished rhombicosidodecahedron**

*height:*  $1/2$   
*circumradius:*  $\sqrt{5+2*\sqrt{5}} = 3.077684$   
*other names:* -  
*comments:* kind of tridiminished icosidodecahedral-cupola (icosidodecahedron as "icosidodecahedron - 3 pentagons" and tridiminished rhombicosidodecahedron (J83) as "rhombicosidodecahedron - 3 pentagonal cupolae")  
*cells:*  $1+1+3$  octahedra +  $3+3+3+3+6+6+6$  square pyramids +  $3+3+3$  pentagonal antiprisms + 1 icosidodecahedron + 3 pentagonal cupolae + 1 tridiminished rhombicosidodecahedron

#### **4.137 icosahedron || icosidodecahedron**

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*circumradius:*  $\sqrt{5+2*\sqrt{5}} = 3.077684$   
*other names:* icosahedral cupola, icosahedral monostratic cup of rectified hexacosichoron  
*cells:*  $20$  octahedra + 1 icosahedron + 12 pentagonal pyramids + 1 icosidodecahedron

#### **4.138 gyroelongated pentagonal pyramid || icosidodecahedron**

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*circumradius:*  $\sqrt{5+2*\sqrt{5}} = 3.077684$   
*other names:* -  
*comments:* kind of diminished icosahedral-cupola (gyroelongated pentagonal pyramid (J11) as "icosahedron - pentagonal pyramid" and icosidodecahedron as "icosidodecahedron - pentagon")  
*cells:*  $5+5+5$  octahedra + 5 square pyramids + 1 gyroelongated pentagonal pyramid +  $1+5+5$  pentagonal pyramids + 1 pentagonal prism + 1 icosidodecahedron

#### **4.139 pentagonal pyramid || rotunda**

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*circumradius:*  $\sqrt{5+2*\sqrt{5}} = 3.077684$   
*other names:* -  
*comments:* kind of diminished icosahedral-cupola (pentagonal pyramid as "icosahedron - gyroelongated pentagonal pyramid (J11)" and rotunda as "icosidodecahedron - rotunda")  
*cells:*  $5$  octahedra + 5 square pyramids +  $1+1+5$  pentagonal pyramids + 1 rotunda + 1 pentagonal cupola

#### **4.140 gyroelongated pentagonal pyramid || rotunda**

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*circumradius:*  $\sqrt{5+2*\sqrt{5}} = 3.077684$   
*other names:* -

*comments:* kind of diminished icosahedral-cupola (gyroelongated pentagonal pyramid (J11) as "icosahedron - pentagonal pyramid" and rotunda as "icosidodecahedron - rotunda")  
*cells:*  $5+5$  octahedra + 5 square pyramids + 1 gyroelongated pentagonal pyramid +  $1+5$  pentagonal pyramids + 1 rotunda + 1 pentagonal cupola

#### **4.141 pentagon || pentagonal pyramid**

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*shear:*  $\sqrt{(5+2*\sqrt{5})/5} = 1.376382$

#### **4.141.1 point || pentagonal prism**

*height:*  $\sqrt{(5-2*\sqrt{5})/20} = 0.162460$   
*shear:* 0  
*circumradius:*  $\sqrt{5+2*\sqrt{5}} = 3.077684$   
*other names:* pentagonal-prismatic pyramid, pentagonal pentagonal-pyramidal wedge  
*comments:* kind of diminished icosahedral cupola (pentagonal pyramid as "icosahedron - gyroelongated pentagonal pyramid (J11)" and pentagon as "icosidodecahedron - icosidodecahedron")  
*cells:*  $5$  square pyramids + 2 pentagonal pyramids + 1 pentagonal prism

#### **4.142 pentagonal antiprism || icosidodecahedron**

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*circumradius:*  $\sqrt{5+2*\sqrt{5}} = 3.077684$   
*other names:* -  
*comments:* kind of bidiminished icosahedral-cupola (pentagonal antiprism as "icosahedron - 2 pentagonal pyramids" and icosidodecahedron as "icosidodecahedron - 2 pentagons")  
*cells:*  $10$  octahedra + 10 square pyramids + 10 pentagonal pyramids + 1 pentagonal antiprism + 2 pentagonal prisms + 1 icosidodecahedron

#### **4.143 metabidiminished icosahedron || icosidodecahedron**

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*circumradius:*  $\sqrt{5+2*\sqrt{5}} = 3.077684$   
*other names:* -  
*comments:* kind of bidiminished icosahedral-cupola (metabidiminished icosahedron (J62) as "icosahedron - 2 pentagonal pyramids" and icosidodecahedron as "icosidodecahedron - 2 pentagons")  
*cells:*  $2+2+2+4$  octahedra +  $2+4+4$  square pyramids +  $2+2+2+4$  pentagonal pyramids + 1 metabidiminished icosahedron + 1 pentagonal prism + 1 icosidodecahedron

#### **4.144 pentagonal antiprism || rotunda**

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*circumradius:*  $\sqrt{5+2*\sqrt{5}} = 3.077684$   
*other names:* -  
*comments:* kind of bidiminshed icosahedral-cupola (pentagonal antiprism as "icosahedron - 2 pentagonal pyramids" and rotunda as "icosidodecahedron - rotunda - pentagon")  
*cells:* 5 octahedra + 5+5 square pyramids + 5 pentagonal pyramids + 1 pentagonal antiprism + 1 pentagonal prism + 1 rotunda + 1 pentagonal cupola

#### **4.145 metabidiminshed icosahedron || rotunda**

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*circumradius:*  $\sqrt{5+2*\sqrt{5}} = 3.077684$   
*other names:* -  
*comments:* kind of bidiminshed icosahedral-cupola (metabidiminshed icosahedron (J62) as "icosahedron - 2 pentagonal pyramids" and rotunda as "icosidodecahedron - rotunda - pentagon")  
*cells:* 1+1+2+2 octahedra + 2+2+2+2 square pyramids + 1+2+2 pentagonal pyramids + 1 metabidiminshed icosahedron + 1 pentagonal prism + 1 rotunda + 1 pentagonal cupola

#### **4.146 pentagon || rotunda**

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*shear:*  $\sqrt{(5+\sqrt{5})}/40 = 0.425325$   
*circumradius:*  $\sqrt{5+2*\sqrt{5}} = 3.077684$   
*other names:* pentagonal rotundaic wedge  
*comments:* kind of bidiminshed icosahedral-cupola (pentagon as "icosahedron - gyroelongated pentagonal pyramid - pentagonal pyramid" and rotunda as "icosidodecahedron - rotunda - pentagon")  
*cells:* 5+5 square pyramids + 5 pentagonal pyramids + 1 pentagonal prism + 1 rotunda + 1 pentagonal cupola

#### **4.147 tridiminshed icosahedron || icosidodecahedron**

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*circumradius:*  $\sqrt{5+2*\sqrt{5}} = 3.077684$   
*other names:* -  
*comments:* kind of tridiminshed icosahedral-cupola (tridiminshed icosahedron (J63) as "icosahedron - 3 pentagonal pyramids" and icosidodecahedron as "icosidodecahedron - 3 pentagons")  
*cells:* 1+1+3 octahedra + 3+6+6 square pyramids + 3+3+3 pentagonal pyramids + 1 tridiminshed icosahedron + 3 pentagonal prisms + 1 icosidodecahedron

#### **4.148 tridiminshed icosahedron || rotunda**

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*circumradius:*  $\sqrt{5+2*\sqrt{5}} = 3.077684$   
*other names:* -  
*comments:* kind of tridiminshed icosahedral-cupola (tridiminshed icosahedron (J63) as "icosahedron - 3 pentagonal pyramids" and rotunda as "icosidodecahedron - rotunda - 2 pentagons")  
*cells:* 1+1 octahedra + 1+2+2+2+2 square pyramids + 1+1+2 pentagonal pyramids + 1 tridiminshed icosahedron + 2 pentagonal prisms + 1 rotunda + 1 pentagonal cupola

#### **4.149 truncated octahedron || truncated cuboctahedron**

*height:* 1/2  
*circumradius:*  $\sqrt{8+3*\sqrt{2}} = 3.498949$   
*other names:* truncated-octahedral monostratic cup of runcitruncated icositetraharon, truncated-octahedral monostratic cup of prismatorhombated icositetraharon  
*cells:* 12 trigonal prisms + 8 hexagonal prisms + 6 square cupolae + 1 truncated octahedron + 1 truncated cuboctahedron

#### **4.150 truncated icosidodecahedron || truncated icosidodecahedron**

*height:* 1  
*circumradius:*  $\sqrt{8+3*\sqrt{5}} = 3.835128$   
*other names:* truncated-icosidodecahedral prism, great-rhombicosidodecahedral prism  
*comments:* uniform  
*cells:* 30 cubes + 20 hexagonal prisms + 12 decagonal prisms + 2 truncated icosidodecahedra

#### **4.151 truncated icosahedron || truncated dodecahedron**

*height:* 1/2  
*circumradius:*  $\sqrt{8+3*\sqrt{5}} = 3.835128$   
*other names:* -  
*cells:* 30 tetrahedra + 20 trigonal cupolae + 12 pentagonal cupolae + 1 truncated icosahedron + 1 truncated dodecahedron

#### **4.152 dodecahedron || rhombicosidodecahedron**

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*circumradius:*  $3+\sqrt{5} = 5.236068$   
*other names:* dodecahedral monostratic cup of runcinated hecatonicosachoron, dodecahedral monostratic cup of runcinated hexacosichoron, dodecahedral monostratic cup of small diprismatohexacosihecatonicosachoron  
*cells:* 20 tetrahedra + 30 trigonal prisms + 12 pentagonal prisms + 1 dodecahedron + 1 rhombicosidodecahedron

#### 4.153 dodecahedron || diminished rhombicosidodecahedron

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*circumradius:*  $3+\sqrt{5} = 5.236068$   
*other names:* -  
*comments:* kind of diminished dodecahedral-monostratic-cup-of-runcinated-hecatonicosachoron (dodecahedron as "dodecahedron - pentagon" and diminished rhombicosidodecahedron (J76) as "rhombicosidodecahedron - pentagonal cupola")  
*cells:* 5+5+5 tetrahedra + 5+5+5+10 trigonal prisms + 1+5+5+5 pentagonal prisms + 1 dodecahedron + 1 diminished rhombicosidodecahedron + 1 pentagonal cupola

#### 4.154 pentagon || pentagonal cupola

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*shear:*  $\sqrt{((25+11*\sqrt{5}))/40} = 1.113516$

#### 4.154.1 decagon || pentagonal prism

*height:*  $\sqrt{((5-2*\sqrt{5}))/20} = 0.162460$   
*shear:* 0  
*circumradius:*  $3+\sqrt{5} = 5.236068$   
*other names:* pentagonal pentagonal-cupolaic wedge  
*comments:* kind of diminished dodecahedral-monostratic-cup-of-runcinated-hecatonicosachoron (pentagon as "dodecahedron - dodecahedron" and pentagonal cupola as "rhombicosidodecahedron - diminished rhombicosidodecahedron (J76)")  
*cells:* 5 tetrahedra + 5 trigonal prisms + 1 pentagonal prism + 2 pentagonal cupolae

#### 4.155 dodecahedron || parabidiminished rhombicosidodecahedron

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*circumradius:*  $3+\sqrt{5} = 5.236068$   
*other names:* -  
*comments:* kind of bidiminished dodecahedral-monostratic-cup-of-runcinated-hecatonicosachoron (dodecahedron as "dodecahedron - 2 pentagons" and parabidiminished rhombicosidodecahedron (J80) as "rhombicosidodecahedron - 2 pentagonal cupolae")  
*cells:* 10 tetrahedra + 10+10 trigonal prisms + 10 pentagonal prisms + 1 dodecahedron + 2 pentagonal cupolae + 1 parabidiminished rhombicosidodecahedron

#### 4.156 dodecahedron || metabidiminished rhombicosidodecahedron

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*circumradius:*  $3+\sqrt{5} = 5.236068$   
*other names:* -  
*comments:* kind of bidiminished dodecahedral-monostratic-cup-of-runcinated-hecatonicosachoron (dodecahedron as "dodecahedron - 2 pentagons" and metabidiminished rhombicosidodecahedron (J81) as "rhombicosidodecahedron - 2 pentagonal cupolae")  
*cells:* 2+2+2+4 tetrahedra + 1+1+2+4+4+4+4 trigonal prisms + 2+2+2+4 pentagonal prisms + 1 dodecahedron + 2 pentagonal cupolae + 1 metabidiminished rhombicosidodecahedron

#### 4.157 dodecahedron || tridiminished rhombicosidodecahedron

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*circumradius:*  $3+\sqrt{5} = 5.236068$   
*other names:* -  
*comments:* kind of tridiminished dodecahedral-monostratic-cup-of-runcinated-hecatonicosachoron (dodecahedron as "dodecahedron - 3 pentagons" and tridiminished rhombicosidodecahedron (J83) as "rhombicosidodecahedron - 3 pentagonal cupolae")  
*cells:* 1+1+3 tetrahedra + 3+3+3+6 trigonal prisms + 3+3+3 pentagonal prisms + 1 dodecahedron + 3 pentagonal cupolae + 1 tridiminished rhombicosidodecahedron

#### 4.158 icosidodecahedron || truncated icosahedron

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*circumradius:*  $\sqrt{(19+8*\sqrt{5})} = 6.073594$   
*other names:* icosidodecahedral monostratic cup of cantellated hexacosichoron, icosidodecahedral monostratic cup of small rhombated hexacosichoron  
*cells:* 12 pentagonal prisms + 20 trigonal cupolae + 1 icosidodecahedron + 1 truncated icosahedron

#### 4.159 rhombicosidodecahedron || truncated dodecahedron

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*circumradius:*  $\sqrt{(23+10*\sqrt{5})} = 6.735034$   
*other names:* rhombicosidodecahedral monostratic cup of cantellated hecatonicosachoron, rhombicosidodecahedral monostratic cup of small rhombated hecatonicosachoron  
*cells:* 20 octahedra + 30 trigonal prisms + 1 rhombicosidodecahedron + 12 pentagonal cupolae + 1 truncated dodecahedron

#### 4.160 gyrated

##### **rhombicosidodecahedron || truncated dodecahedron**

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*circumradius:*  $\sqrt{23+10*\sqrt{5}} = 6.735034$   
*other names:* -  
*comments:* kind of gyrated  
 rhombicosidodecahedral-monostratic-cup-of-cantellated-hecatonicosachoron (gyrated rhombicosidodecahedron (J72) as "diminished rhombicosidodecahedron (J76) + pentagonal cupola" and truncated dodecahedron as "truncated dodecahedron + decagon" (see 4.164, 4.165) joining at the decagonal prism)  
*cells:* 5+5+5 octahedra + 5+5 square pyramids + 5+5+5+10 trigonal prisms + 1 gyrated rhombicosidodecahedron + 1+1+5+5 pentagonal cupolae + 1 truncated dodecahedron

#### 4.161 parabigyrate

##### **rhombicosidodecahedron || truncated dodecahedron**

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*circumradius:*  $\sqrt{23+10*\sqrt{5}} = 6.735034$   
*other names:* -  
*comments:* kind of bigyrate  
 rhombicosidodecahedral-monostratic-cup-of-cantellated-hecatonicosachoron (parabigyrate rhombicosidodecahedron (J73) as "parabidiminished rhombicosidodecahedron (J80) + 2 pentagonal cupolae" and truncated dodecahedron as "truncated dodecahedron + 2 decagons" (see 4.169, 4.165) joining at the decagonal prism)  
*cells:* 10 octahedra + 10+10 square pyramids + 10+10+10 trigonal prisms + 1 parabigyrate rhombicosidodecahedron + 2+10 pentagonal cupolae + 1 truncated dodecahedron

#### 4.162 metabigyrate

##### **rhombicosidodecahedron || truncated dodecahedron**

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*circumradius:*  $\sqrt{23+10*\sqrt{5}} = 6.735034$   
*other names:* -  
*comments:* kind of bigyrate  
 rhombicosidodecahedral-monostratic-cup-of-cantellated-hecatonicosachoron (metabigyrate rhombicosidodecahedron (J74) as "metabidiminished rhombicosidodecahedron (J81) + 2 pentagonal cupolae" and truncated dodecahedron as "truncated dodecahedron + 2 decagons" (see 4.170, 4.165) joining at the decagonal prism)  
*cells:* 2+2+2+4 octahedra + 2+2+4+4+4+4 square pyramids + 1+1+2+2+4+4+4+4+4+4 trigonal prisms + 1 metabigyrate rhombicosidodecahedron + 2+2+2+2+4 pentagonal cupolae + 1 truncated dodecahedron

#### 4.163 trigyrate

##### **rhombicosidodecahedron || truncated dodecahedron**

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*circumradius:*  $\sqrt{23+10*\sqrt{5}} = 6.735034$   
*other names:* -  
*comments:* kind of trigyrate  
 rhombicosidodecahedral-monostratic-cup-of-cantellated-hecatonicosachoron (trigyrate rhombicosidodecahedron (J75) as "tridiminished rhombicosidodecahedron (J83) + 3 pentagonal cupolae" and truncated dodecahedron as "truncated dodecahedron + 3 decagons" (see 4.172, 4.165) joining at the decagonal prism)  
*cells:* 1+1+3 octahedra + 3+3+6+6+6+6 square pyramids + 3+3+3+3+6+6+6 trigonal prisms + 1 trigyrate rhombicosidodecahedron + 3+3+3+3 pentagonal cupolae + 1 truncated dodecahedron

#### 4.164 diminished

##### **rhombicosidodecahedron || truncated dodecahedron**

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*circumradius:*  $\sqrt{23+10*\sqrt{5}} = 6.735034$   
*other names:* -  
*comments:* kind of diminished  
 rhombicosidodecahedral-monostratic-cup-of-cantellated-hecatonicosachoron (diminished rhombicosidodecahedron (J76) as "rhombicosidodecahedron - pentagonal cupola" and truncated dodecahedron as "truncated dodecahedron - decagon")  
*cells:* 5+5+5 octahedra + 5 square pyramids + 5+5+5+10 trigonal prisms + 1 decagonal prism + 1 diminished rhombicosidodecahedron + 1+5+5 pentagonal cupolae + 1 truncated dodecahedron

#### 4.165 decagon || pentagonal cupola

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*shear:*  $\sqrt{(25+11*\sqrt{5})/8} = 2.489893$

##### 4.165.1 pentagon || decagonal prism

*height:*  $\sqrt{(5-2*\sqrt{5})/20} = 0.162460$   
*shear:* 0  
*circumradius:*  $\sqrt{23+10*\sqrt{5}} = 6.735034$   
*other names:* decagonal pentagonal-cupolae wedge  
*comments:* kind of diminished  
 rhombicosidodecahedral-monostratic-cup-of-cantellated-hecatonicosachoron (pentagonal cupola as "rhombicosidodecahedron - diminished rhombicosidodecahedron (J76)" and decagon as "truncated dodecahedron - truncated dodecahedron")  
*cells:* 5 square pyramids + 5 trigonal prisms + 1 decagonal prism + 2 pentagonal cupolae

**4.166 gyrated paradiminished rhombicosidodecahedron || truncated dodecahedron**

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*circumradius:*  $\sqrt{23+10*\sqrt{5}} = 6.735034$   
*other names:* -  
*comments:* kind of gyrated rhombicosidodecahedral-monostratic-cup-of-cantellated-hecatonicosachoron (gyrated paradiminished rhombicosidodecahedron (J77) as "parabidiminished rhombicosidodecahedron (J80) + pentagonal cupola" and truncated dodecahedron as "truncated dodecahedron + decagon" (see 4.169, 4.165) joining at a decagonal prism) -resp.- kind of diminished rhombicosidodecahedral-monostratic-cup-of-cantellated-hecatonicosachoron (gyrated paradiminished rhombicosidodecahedron (J77) as "gyrated rhombicosidodecahedron (J72) - pentagonal cupola" and truncated dodecahedron as "truncated dodecahedron - decagon")  
*cells:* 5+5 octahedra + 5+5+5 square pyramids + 5+5+5+10 trigonal prisms + 1+1+5 pentagonal cupolae + 1 gyrated paradiminished rhombicosidodecahedron + 1 truncated dodecahedron

**4.167 gyrated metadiminished rhombicosidodecahedron || truncated dodecahedron**

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*circumradius:*  $\sqrt{23+10*\sqrt{5}} = 6.735034$   
*other names:* -  
*comments:* kind of gyrated rhombicosidodecahedral-monostratic-cup-of-cantellated-hecatonicosachoron (gyrated metadiminished rhombicosidodecahedron (J78) as "metabidiminished rhombicosidodecahedron (J81) + pentagonal cupola" and truncated dodecahedron as "truncated dodecahedron + decagon" (see 4.170, 4.165) joining at a decagonal prism) -resp.- kind of diminished rhombicosidodecahedral-monostratic-cup-of-cantellated-hecatonicosachoron (gyrated metadiminished rhombicosidodecahedron (J78) as "gyrated rhombicosidodecahedron (J72) - pentagonal cupola" and truncated dodecahedron as "truncated dodecahedron - decagon")  
*cells:* 1+1+2+2+2+2 octahedra + 1+1+1+2+2+2+2+2 square pyramids + 1+1+1+2+2+2+2+2+2+2+2+2+2+2 trigonal prisms + 1+1+1+2+2+2+2 pentagonal cupolae + 1 gyrated metadiminished rhombicosidodecahedron + 1 truncated dodecahedron

**4.168 bigyrated diminished rhombicosidodecahedron || truncated dodecahedron**

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*circumradius:*  $\sqrt{23+10*\sqrt{5}} = 6.735034$   
*other names:* -

*comments:* kind of bigyrated rhombicosidodecahedral-monostratic-cup-of-cantellated-hecatonicosachoron (bigyrated diminished rhombicosidodecahedron (J79) as "tridiminished rhombicosidodecahedron (J83) + 2 pentagonal cupolae" and truncated dodecahedron as "truncated dodecahedron + 2 decagons" (see 4.172, 4.165) joining at decagonal prisms) -resp.- kind of diminished rhombicosidodecahedral-monostratic-cup-of-cantellated-hecatonicosachoron (bigyrated diminished rhombicosidodecahedron (J79) as "metabigyrate rhombicosidodecahedron (J74) - pentagonal cupola" and truncated dodecahedron as "truncated dodecahedron - decagon")  
*cells:* 1+1+1+2 octahedra + 1+2+2+2+2+2+2+2+2+2+2+2+2+2+2 square pyramids + 1+1+1+2+2+2+2+2+2+2+2+2+2+2+2 trigonal prisms + 1+1+1+2+2+2+2 pentagonal cupolae + 1 bigyrated diminished rhombicosidodecahedron + 1 truncated dodecahedron

**4.169 parabidiminished rhombicosidodecahedron || truncated dodecahedron**

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*circumradius:*  $\sqrt{23+10*\sqrt{5}} = 6.735034$   
*other names:* -  
*comments:* kind of bidiminished rhombicosidodecahedral-monostratic-cup-of-cantellated-hecatonicosachoron (parabidiminished rhombicosidodecahedron (J80) as "rhombicosidodecahedron - 2 pentagonal cupolae" and truncated dodecahedron as "truncated dodecahedron - 2 decagons")  
*cells:* 10 octahedra + 10 square pyramids + 10+10 trigonal prisms + 2 decagonal prisms + 10 pentagonal cupolae + 1 parabidiminished rhombicosidodecahedron + 1 truncated dodecahedron

**4.170 metabidiminished rhombicosidodecahedron || truncated dodecahedron**

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*circumradius:*  $\sqrt{23+10*\sqrt{5}} = 6.735034$   
*other names:* -  
*comments:* kind of bidiminished rhombicosidodecahedral-monostratic-cup-of-cantellated-hecatonicosachoron (metabidiminished rhombicosidodecahedron (J81) as "rhombicosidodecahedron - 2 pentagonal cupolae" and truncated dodecahedron as "truncated dodecahedron - 2 decagons")  
*cells:* 2+2+2+4 octahedra + 2+4+4 square pyramids + 1+1+2+4+4+4+4 trigonal prisms + 2 decagonal prisms + 2+2+2+4 pentagonal cupolae + 1 metabidiminished rhombicosidodecahedron + 1 truncated dodecahedron

**4.171 gyrated bidiminished rhombicosidodecahedron || truncated dodecahedron**

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*circumradius:*  $\sqrt{23+10*\sqrt{5}} = 6.735034$   
*other names:* -  
*comments:* kind of gyrated rhombicosidodecahedral-monostratic-cup-of-cantellated-hecatonicosachoron (gyrated bidiminished rhombicosidodecahedron (J82) as "tridiminished rhombicosidodecahedron (J83) + pentagonal cupola" and truncated dodecahedron as "truncated dodecahedron + decagon" (see 4.172, 4.165) joining at a decagonal prism) -resp.- kind of bidiminished rhombicosidodecahedral-monostratic-cup-of-cantellated-hecatonicosachoron (gyrated bidiminished rhombicosidodecahedron (J82) as "gyrated rhombicosidodecahedron (J72) - 2 pentagonal cupolae" and truncated dodecahedron as "truncated dodecahedron - 2 decagons")  
*cells:* 1+1+1+2 octahedra + 1+1+2+2+2+2+2+2+2+2 square pyramids + 1+1+1+2+2+2+2+2+2+2+2+2 trigonal prisms + 2 decagonal prisms + 1+1+1+1+2+2+2 pentagonal cupolae + 1 gyrated bidiminished rhombicosidodecahedron + 1 truncated dodecahedron

**4.172 tridiminished rhombicosidodecahedron || truncated dodecahedron**

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*circumradius:*  $\sqrt{23+10*\sqrt{5}} = 6.735034$   
*other names:* -  
*comments:* kind of tridiminished rhombicosidodecahedral-monostratic-cup-of-cantellated-hecatonicosachoron (tridiminished rhombicosidodecahedron (J83) as "rhombicosidodecahedron - 3 pentagonal cupolae" and truncated dodecahedron as "truncated dodecahedron - 3 decagons")  
*cells:* 1+1+3 octahedra + 3+3+6 square pyramids + 3+3+3+6 trigonal prisms + 3 decagonal prisms + 3+3+3 pentagonal cupolae + 1 tridiminished rhombicosidodecahedron + 1 truncated dodecahedron

**4.173 truncated dodecahedron || truncated icosidodecahedron**

*height:*  $(\sqrt{5}-1)/4 = 0.309017$   
*circumradius:*  $\sqrt{48+21*\sqrt{5}} = 9.744610$   
*other names:* truncated-dodecahedral monostratic cup of runcitruncated hecatonicosachoron, truncated-dodecahedral monostratic cup of prismatorhombated hexacosichoron  
*cells:* 30 trigonal prisms + 20 trigonal cupolae + 12 decagonal prisms + 1 truncated dodecahedron + 1 truncated icosidodecahedron

**4.174 n-gon || n-antiprism (n ≠ 2, 3, 4, 5, 6, 8, 10)**

*height:*  $\sqrt{(1+3*\cos(\pi/n))/(2+4*\cos(\pi/n))}$   
*shear:*  $1/\sqrt{8+24*\cos(\pi/n)+16*\cos^2(\pi/n)}$

**4.174.1 n-gon || gyrated n-prism (n ≠ 3, 4, 5, 6, 8, 10)**

*height:*  $\sqrt{(1+3*\cos(\pi/n))/(4+4*\cos(\pi/n))}$   
*shear:* 0  
*circumradius:*  $\sqrt{(1+2*\cos(\pi/n)-2*\cos^2(\pi/n))/(2+4*\cos(\pi/n)-6*\cos^2(\pi/n))}$   
*other names:* general n-gonal n-antiprismatic wedge  
*cells:* n tetrahedra + n square pyramids + 2 n-antiprisms + 1 n-prism

**4.175 n-gon || n-prism (n ≠ 3, 4, 5, 6, 8, 10)**

*height:*  $\sqrt{3/4} = 0.866025$   
*shear:* 0  
*circumradius:*  $\sqrt{(4+3*\csc^2(\pi/n))/12}$   
*other names:* trigon- n-gon -diprism, direct sum of trigon and n-gon, n-gonal n-prismatic wedge  
*comments:* uniform  
*cells:* n trigonal prisms + 3 n-gonal prisms

**4.176 n-gonal antiprism || n-gonal antiprism (n ≠ 2, 3, 4, 5, 6, 8, 10)**

*height:* 1

**4.176.1 n-gonal prism || gyrated n-gonal prism (n ≠ 3, 4, 5, 6, 8, 10)**

*height:*  $\sqrt{(1+2*\cos(\pi/n))/(2+2*\cos(\pi/n))}$   
*circumradius:*  $\sqrt{(5-4*\cos(\pi/n))/(8-8*\cos(\pi/n))}$   
*other names:* general n-gonal-antiprismatic prism  
*comments:* uniform  
*cells:* 2n trigonal prisms + 2 n-gonal antiprisms + 2 n-gonal prisms

**4.177 n-gonal prism || n-gonal prism (n ≠ 3, 4, 5, 6, 8, 10)**

*height:* 1  
*circumradius:*  $\sqrt{2+\csc^2(\pi/n)}/2$   
*other names:* general n-gonal-prismatic prism, square- n-gon -diprism, direct sum of square and n-gon  
*comments:* uniform  
*cells:* n cubes + 4 n-gonal prisms

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tid	truncated dodecahedron
toe	truncated octahedron
trig	see 3g
tut	truncated tetrahedron
dip	diprism, duoprism
hex	hexadecachoron
ico	icositetrachoron



abriv.	Name
pen	pentachoron
prico	runcitruncated icositetrachoron, prismatorhombated icositetrachoron
prip	runcitruncated pentachoron, prismatorhombated pentachoron
prix	runcitruncated hecatonicosachoron, prismatorhombated hexacosichoron
proh	runcitruncated tesseract, runcitruncated octachoron, prismatorhombated hexadecachoron
rap	rectified pentachoron
rico	rectified icositetrachoron
rit	rectified tesseract, rectified octachoron
rox	rectified hexacosichoron
sidpith	runcinated tesseract, runcinated octachoron, runcinated hexadecachoron, small diprismatotesseractihexadecachoro n
sidpixhi	runcinated hecatonicosachoron, runcinated hexacosichoron, small

abriv.	Name
	diprismatohexacosihecatonicosacho ron
spic	runcinated icositetrachoron, small prismatotetracontaocotachoron
spid	runcinated pentachoron,(small) prismatodecachoron
srahi	cantellated hecatonicosachoron, small rhombated hecatonicosachoron
srico	cantellated icositetrachoron, small rhombated icositetrachoron
srip	cantellated pentachoron, small rhombated pentachoron, (small) prismatodispentachoron
srit	cantellated tesseract, cantellated octachoron, small rhombated tesseract
srix	cantellated hexacosichoron, small rhombated hexacosichoron
tes	tesseract, octachoron

**Table 3: some abbreviations for facets and polychora**

This index references to the list, again using the form "x || y", but this time it uses abbreviations for the top and bottom facets (essentially the numbers of the Johnson solids respectively most of the shortnames introduced by J. Bowers, see Table 3). Further this listing is completely lexicographic.

10ap    10ap	4.96	3g    gyro 3p	4.6.2	4g    4g	3.6
10ap    10g	4.93	3g    gyro tet	4.3.1	4g    4p	4.18.1
10g    10g	3.17	3g    incl 3g	4.4.2	4g    4pyr	4.26
10g    10p	4.94	3g    J63	4.33	4g    8g	3.15
10g    5ap	4.133.1	3g    line	3.3	4g    8p	4.105.1
10g    5cup	4.165	3g    oct	4.6.1	4g    co	4.28
10g    5g	3.18	3g    ortho 4g	4.7.3	4g    dual 4g	3.5
10g    5p	4.154.1	3g    perp line	4.1.1	4g    gyro 4pyr	4.17
10g    dual 10g	3.16	3g    point	3.1	4g    gyro 4p	4.14.1
10g    gyro 10p	4.93.1	3g    tet	4.7.1	4g    line	3.4.1
10p    10p	4.97	3p    3p	4.18	4g    ortho 4g	4.9.2
10p    5g	4.165.1	3p    4g	4.12.1	4g    perp line	4.4.3
10p    5p	4.117.1	3p    6g	4.25.1	4g    point	3.3
10p    gyro 10p	4.96.1	3p    6p	4.45.1	4g    tet	4.8.1
3cup    3cup	4.45	3p    gyro 3p	4.11.1	4p    4p	4.20
3cup    3g	4.24	3p    ortho line	4.8.2	4p    8g	4.73.1
3cup    4pyr	4.32	3p    para line	4.9.1	4p    8p	4.69.1
3cup    6g	4.51	3p    point	4.7.2	4p    co	4.35
3cup    gyro 3g	4.27	3p    refl ortho 3p	4.13	4p    gyro 4p	4.19.1
3cup    oct	4.30	4ap    4ap	4.19	4p    gyro 4pyr	4.16
3cup    tet	4.24	4ap    4g	4.14	4p    ike	4.21
3cup    tut	4.50	4ap    8g	4.64.1	4p    J19	4.72
3g    3g	3.4	4ap    point	4.17.1	4p    line	4.12.2
3g    3p	4.10	4cup    4cup	4.69	4p    oct	4.15
3g    4pyr	4.8	4cup    4pyr	4.109	4p    point	4.26.1
3g    6g	3.10	4cup    4g	4.73	4p    sirco	4.71
3g    6p	4.51.1	4cup    8g	4.105	4pyr    4pyr	4.12
3g    dual 3g	3.2	4cup    gyro 4g	4.64	4pyr    co	4.31

4pyr    J19	4.108	doe    doe	4.74	J76    tid	4.164
4pyr    line	4.7	doe    id	4.77	J77    J77	4.118
4pyr    point	4.4	doe    ike	4.78	J77    tid	4.166
4pyr    tet	4.6	doe    J11	4.79	J78    J78	4.119
5ap    5ap	4.39	doe    J62	4.82	J78    tid	4.167
5ap    5g	4.22	doe    J63	4.83	J79    J79	4.120
5ap    doe	4.81	doe    J76	4.153	J79    tid	4.168
5ap    id	4.142	doe    J80	4.155	J80    J80	4.121
5ap    J6	4.144	doe    J81	4.156	J80    tid	4.169
5ap    point	4.80.1	doe    J83	4.157	J81    J81	4.122
5cup    5cup	4.117	doe    srid	4.152	J81    tid	4.170
5cup    5g	4.154	girco    girco	4.125	J82    J82	4.123
5cup    gyro 5g	4.133	girco    tic	4.128	J82    tid	4.171
5g    5g	3.9	girco    toe	4.149	J83    J83	4.124
5g    5p	4.34	grid    grid	4.150	J83    tid	4.172
5g    5pyr	4.141	grid    tid	4.173	line    line	2.2
5g    dual 5g	3.7	id    id	4.90	line    perp line	3.1.1
5g    gyro 5p	4.22.1	id    ike	4.137	line    point	2.1
5g    gyro 5pyr	4.80	id    J11	4.138	line    tet	4.4.1
5g    J6	4.146	id    J62	4.143	n-ap    n-ap	4.176
5g    perp line	4.86.1	id    J63	4.147	n-ap    n-g	4.174
5g    point	3.8	id    J76	4.132	n-g    dual n-g	3.19
5p    5p	4.42	id    J80	4.134	n-g    gyro n-p	4.174.1
5p    gyro 5p	4.39.1	id    J81	4.135	n-g    n-g	3.20
5p    line	4.38.1	id    J83	4.136	n-g    n-p	4.175
5p    point	4.141.1	id    srid	4.131	n-p    gyro n-p	4.176.1
5pyr    5pyr	4.38	id    ti	4.158	n-p    n-p	4.177
5pyr    J6	4.139	ike    ike	4.36	oct    oct	4.11
5pyr    point	4.86	ike    point	4.84	oct    point	4.3
6ap    6ap	4.53	J11    J11	4.37	oct    sirco	4.107
6ap    6g	4.46	J11    J6	4.140	oct    tet	4.5
6g    6g	3.12	J11    point	4.85	oct    tut	4.52
6g    6p	4.47	J19    gyro tic	4.104	point    point	line
6g    dual 6g	3.11	J19    J19	4.68	point    tet	4.1
6g    gyro 6p	4.46.1	J19    tic	4.103	sirco    gyro tic	4.102
6g    oct	4.27.1	J27    J27	4.44	sirco    sirco	4.66
6p    6p	4.54	J27    tut	4.49	sirco    tic	4.100
6p    gyro 6p	4.53.1	J34    J34	4.91	sirco    toe	4.75
8ap    8ap	4.65	J37    J37	4.67	snic    snic	4.60
8ap    8g	4.58	J37    tic	4.101	snid    snid	4.110
8g    8g	3.14	J6    J6	4.92	srid    srid	4.111
8g    8p	4.59	J6    J62	4.145	srid    ti	4.126
8g    dual 8g	3.13	J6    J63	4.148	srid    tid	4.159
8g    gyro 8p	4.58.1	J62    J62	4.40	tet    dual tet	4.2
8p    8p	4.70	J62    point	4.87	tet    tet	4.9
8p    co	4.63	J63    J63	4.41	tet    tut	4.56
8p    gyro 8p	4.65.1	J63    point	4.88	ti    ti	4.127
8p    tic	4.106	J72    J72	4.112	ti    tid	4.151
co    co	4.43	J72    tid	4.160	tic    tic	4.99
co    J19	4.62	J73    J73	4.113	tic    toe	4.98
co    oct	4.29	J73    tid	4.161	tid    tid	4.130
co    sirco	4.61	J74    J74	4.114	toe    toe	4.89
co    tet	4.23	J74    tid	4.162	toe    tut	4.76
co    tic	4.129	J75    J75	4.115	tut    inv tut	4.55
co    toe	4.95	J75    tid	4.163	tut    tut	4.57
co    tut	4.48	J76    J76	4.116		